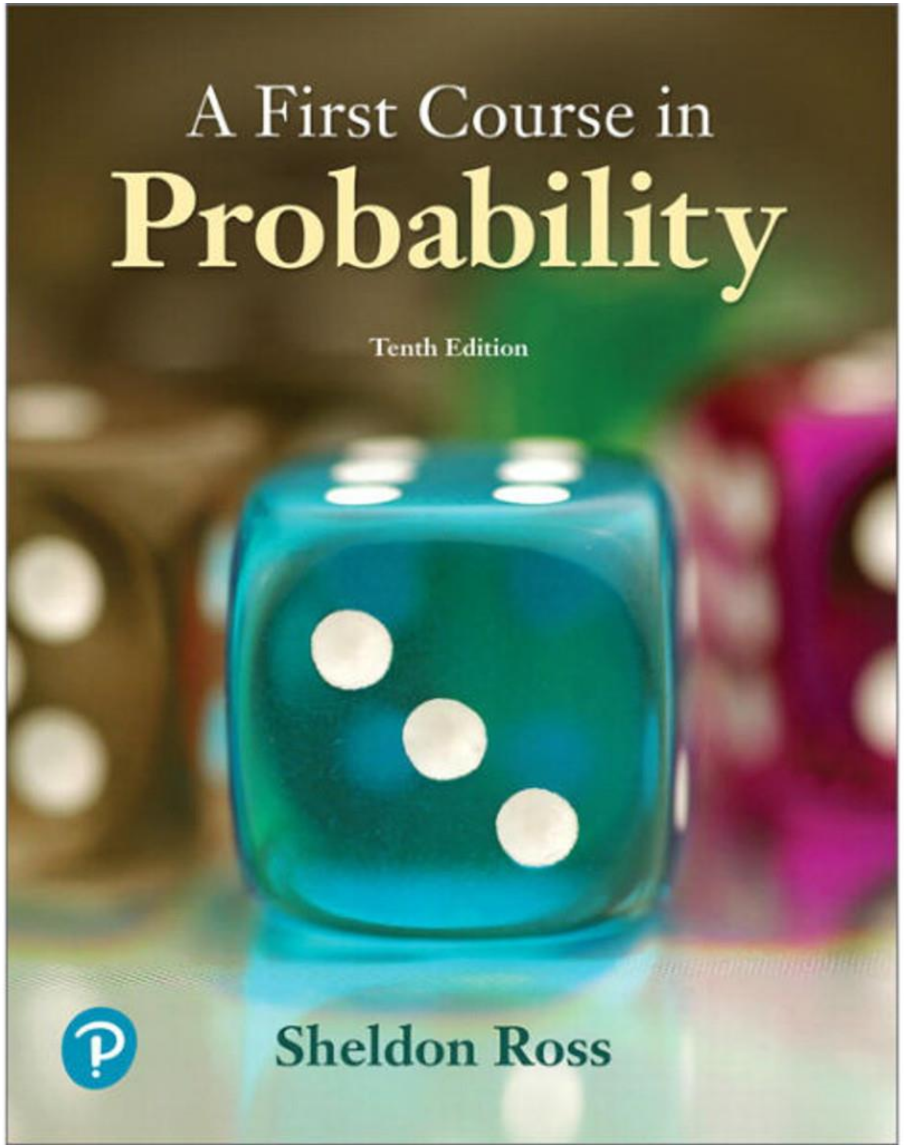
Lecture 9.2

Normal random variables



Today's reading: 5.4

Next class: 5.4.1

HW7 now available. Due Friday

Today's draft problem

To be presented by today's draftee.

The PDF of X is given by

$$f(x) = \begin{cases} a + bx^2, & 0 < x < 1 \\ 0, & else \end{cases}$$

If $E[X] = 3/5$, find a and b.

Normal random variables

Definition (more-or-less from the book):

A continuous $\mathbb{R}$-valued random variable X (on a sample space S with probability measure P…) is called *normal* with parameters μ and σ^2 if its PDF is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

We will show momentarily on the chalkboard why this is a valid PDF, i.e., why

$$\int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \mathrm{dx} = 1$$

Standard normal random variables

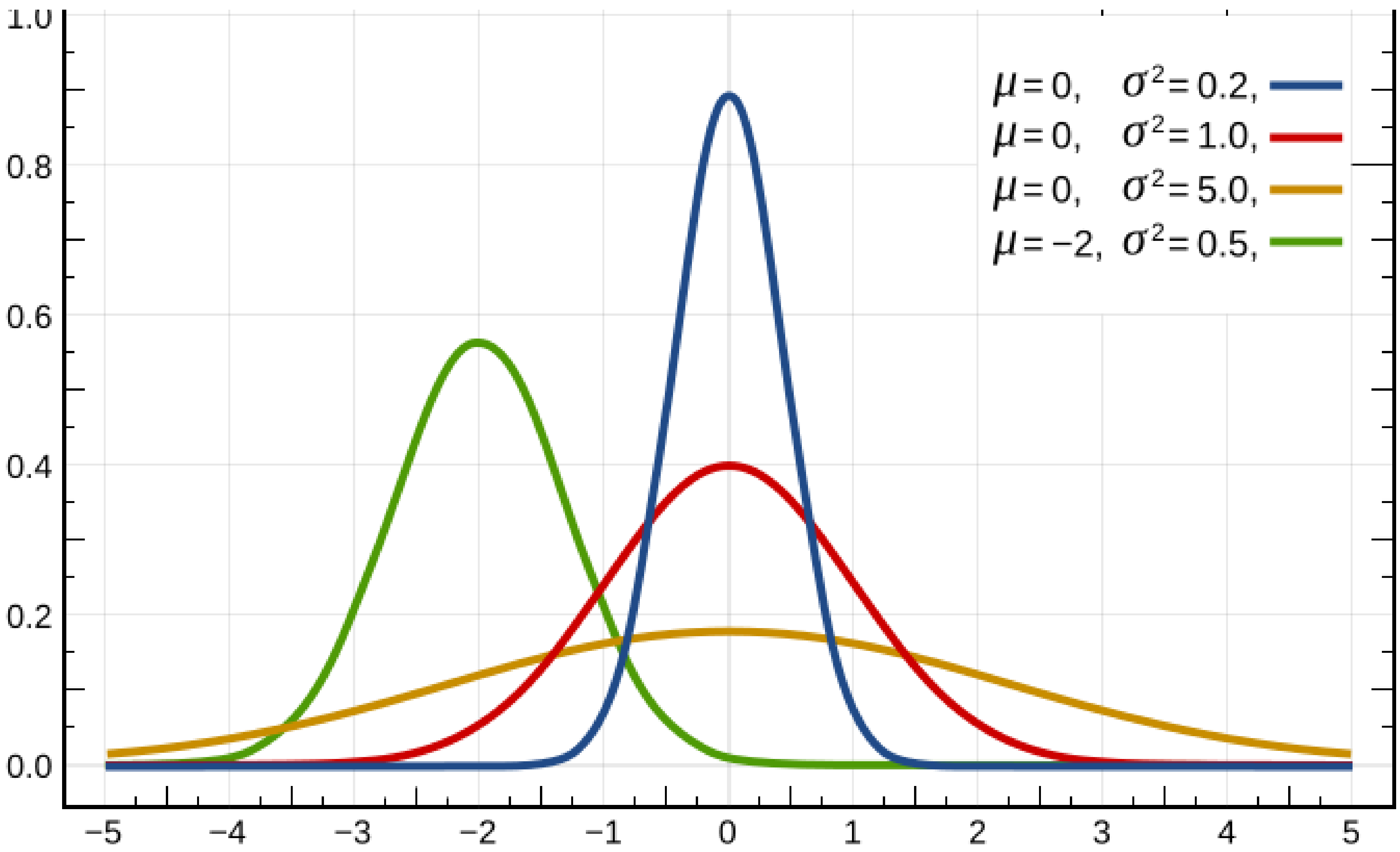
Definition (more-or-less from the book):

A normal random variable is *standard normal* if its parameters are $\mu = 0$ and $\sigma^2 = 1$.

Motivation:

If X is a normal random variable with parameters μ and σ^2, then the random variable $\mathrm{Z} = \frac{X-\mu}{\sigma}$ is standard normal. (Why?)

In particular, it will be fairly easy to transfer most results about the *standard* normal random variable to results for *general* normal random variables.



$\mu=0, \quad \sigma^2=0.2,$
$\mu=0, \quad \sigma^2=1.0,$
$\mu=0, \quad \sigma^2=5.0,$
$\mu=-2, \quad \sigma^2=0.5,$
1.0
0.8
0.6
0.4
0.2
0.0
−5
−4
−3
−2
−1
0
1
2
3
4
5

Properties of normal random variables

Let Z be a standard normal random variable. Then:

- The PDF of Z is a valid PDF.
- $E[Z] = 0$
- $Var(Z) = 1$

Let's justify these on the board.

Now let X be any normal variable with parameters μ and σ^2. Then:

- The PDF of X is a valid PDF.
- $E[X] = \mu$
- $Var(Z) = \sigma^2$

Justifying these is easy now!

CDFs of normal random variables

Let Z be a standard normal random variable. Then:

We denote its CDF by $\Phi(a)$. That is,

$$\Phi(a) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{a} e^{-y^2/2}\, dy$$

Now let X be any normal variable with parameters μ and σ^2.

We can use $\Phi(x)$ to compute the CDF of X, using that

$$\mathrm{F_X(a)} = \mathrm{P\{X} \leq a\} = \int_{-\infty}^{a} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \mathrm{dx} = \int_{-\infty}^{(a-\mu)/\sigma} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy = \Phi\left(\frac{a-\mu}{\sigma}\right)$$

Table 5.1 Area $\Phi(x)$ Under the Standard Normal Curve to the Left of X.

X	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Friday’s draft problem

To be presented by Friday’s draftee.

<u>Variation of Ex. 4d (with more accurate numbers)</u>

An expert witness in a paternity suit testifies that the length (in days) of human gestation is approximately 280 days with a standard deviation of 14 days. The defendant in the suit (a long haul trucker) is able to prove that he was not in the mother’s time-zone for a period of 40 days that began 300 days before the child’s birth.

If the defendant were the father, what is the probability that the mother had the kind of exceptionally long or short gestation period implied by the evidence?