Lecture 9.2

Normal random variables





Today's reading: 5.4

Next class: 5.4.1

HW7 now available. Due Friday



Today's draft problem

To be presented by today's draftee.



The PDF of X is given by $f(x) = \begin{cases} a + bx^2, & 0 < x < 1 \\ 0, & else \end{cases}$ If E[X] = 3/5, find a and b.



Normal random variables

Definition (more-or-less from the book):

A continuous \mathbb{R} -valued random variable X (on a sample space S with probability measure P...) is called <u>normal</u> with parameters μ and σ^2 if its PDF is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

We will show momentarily on the chalkboard why this is a valid PDF, i.e., why

$$\int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} dx = 1$$



<u>Standard</u> normal random variables

Definition (more-or-less from the book):

A normal random variable is <u>standard normal</u> if its parameters are $\mu = 0$ and $\sigma^2 = 1$.

Motivation:

If X is a normal random variable with parameters μ and σ^2 , then the random variable Z = $\frac{X-\mu}{\sigma}$ is standard normal. (Why?)

In particular, it will be fairly easy to transfer most results about the *standard* normal random variable to results for *general* normal random variables.





Properties of normal random variables

Let Z be a standard normal random variable. Then:

- The PDF of Z is a valid PDF.
- E[Z] = 0
- Var(Z) = 1

Let's justify these on the board.

Now let *X* be any normal variable with parameters μ and σ^2 . Then:

• The PDF of *X* is a valid PDF.

• $E[X] = \mu$

• $Var(Z) = \sigma^2$

Justifying these is easy now!



CDFs of normal random variables

Let Z be a standard normal random variable. Then:

We denote its CDF by $\Phi(a)$. That is,

$$\Phi(a) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{a} e^{-y^2/2} \, dy$$

Now let X be any normal variable with parameters μ and σ^2 . We can use $\Phi(x)$ to compute the CDF of X, using that

$$F_{X}(a) = P\{X \le a\} = \int_{-\infty}^{a} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^{2}/2\sigma^{2}} dx = \int_{-\infty}^{(a-\mu)/\sigma} \frac{1}{\sqrt{2\pi}} e^{-y^{2}/2} dy = \Phi\left(\frac{a-\mu}{\sigma}\right)$$



1 10 5000 5040 5080 5120 5160 5199 5239 5279 5319 1 5398 5438 5478 5517 5557 5596 5636 5675 5714 1 5393 5832 5871 5910 5948 5987 6026 6064 6103 2 5793 5832 5871 5910 5948 5987 6026 6064 6443 6484 4 6554 6591 6628 6664 6700 6736 6772 6808 6844 5 6915 6950 .6985 .7019 .7054 .7082 .7157 .7190 7 7580 .7611 .7642 .7673 .7704 .7774 .7784 .8781 .8179 .8123 .8315 .8340 .8365 10 .8413 .8438 .8461 .8485 .8508 .8531 .8574 .8577 .8599 <td< th=""><th>Tal X</th><th>00</th><th>.01</th><th>.02</th><th>.03</th><th>.04</th><th>.05</th><th>.06</th><th>.07</th><th>.08</th><th>.09</th></td<>	Tal X	00	.01	.02	.03	.04	.05	.06	.07	.08	.09
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2 5/93 5/832 5/811 5/91 5/91 6/331 6/368 6/406 6/443 6/443 6/443 3 6/179 6/217 6/225 6/293 6/331 6/368 6/772 6/808 6/844 4 6/554 6/591 6/225 6/293 6/331 6/368 7/123 7/157 7/190 5 6/915 6/950 6/985 7/019 7/04 7/734 7/764 7/794 7/823 6 7/257 7/291 7/324 7/357 7/389 7/422 7/454 7/486 7/517 7 7/580 7/611 7/642 7/673 7/704 7/734 7/764 7/794 7/823 8 7/881 7910 7/939 7/967 7/995 8/023 8/051 8/365 8/366 10 .8413 .8488 8/485 8/508 8/511 8/577 8/599 11 .8643 .8665 .8668 .8708 8/729 8/719 .8770 8/790 8/810	.1	.5398	.5438	.5470	5010	5948	.5987	.6026	.6064	.6103	.614
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1.3.9032.9049.9066.9082.9099.9115.9131.9147.91621.4.9192.9207.9222.9236.9251.9265.9279.9292.93061.5.9332.9345.9357.9370.9382.9394.9406.9418.94291.6.9452.9463.9474.9484.9495.9505.9515.9525.9535.7.9554.9564.9573.9582.9591.9599.9608.9616.9625.8.9641.9649.9656.9664.9671.9678.9686.9693.9699.9.9713.9719.9726.9732.9738.9744.9750.9756.9761.0.9772.9778.9783.9788.9793.9798.9803.9808.9812.1.9821.9826.9830.9834.9838.9842.9846.9850.9854.2.9861.9864.9868.9871.9875.9878.9881.9884.9887.3.9893.9896.9898.9901.9904.9906.9909.9911.9913.5.9938.9940.9941.9943.9945.9946.9948.9949.9951.7.9965.9966.9957.9959.9960.9961.9962.9963.8.9974.9975.9976.9977.9977.9978.9979.9979.9980	.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.901
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5 9332 9345 9357 9370 9382 9394 9406 9418 9429 6 9452 9463 9474 9484 9495 9505 9515 9525 9535 7 9554 9564 9573 9582 9591 9599 9608 9616 9625 8 9641 9649 9656 9664 9671 9678 9686 9693 9699 9 9713 9719 9726 9732 9738 9744 9750 9756 9761 0 9772 9778 9783 9788 9793 9798 9803 9808 9812 1 9821 9826 9830 9834 9838 9842 9846 9850 9854 2 9861 9864 9868 9871 9875 9878 9881 9884 9887 3 9896 9898 9901 9904 9906 9909 9911 9913 3 9920 9922 9925 9927 9929 9931 9932 9934 4 9918 9920 9922 9925 9927 9929 9931 9932 9934 6 9953 9955 9956 9957 9959 9960 9961 9962 9963 7 9965 9966 9967 9977 9977 9977 9979 9979 9981 9982 9987	.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.931
.6 $.9452$ $.9463$ $.9474$ $.9484$ $.9495$ $.9505$ $.9515$ $.9525$ $.9535$ $.7$ $.9554$ $.9564$ $.9573$ $.9582$ $.9591$ $.9599$ $.9608$ $.9616$ $.9625$ $.8$ $.9641$ $.9649$ $.9656$ $.9664$ $.9671$ $.9678$ $.9686$ $.9693$ $.9699$ $.9$ $.9713$ $.9719$ $.9726$ $.9732$ $.9738$ $.9744$ $.9750$ $.9756$ $.9761$ $.0$ $.9772$ $.9778$ $.9783$ $.9788$ $.9793$ $.9798$ $.9803$ $.9808$ $.9812$ $.1$ $.9821$ $.9826$ $.9830$ $.9834$ $.9838$ $.9842$ $.9846$ $.9850$ $.9854$ $.2$ $.9861$ $.9864$ $.9868$ $.9871$ $.9875$ $.9878$ $.9881$ $.9884$ $.9887$ $.3$ $.9893$ $.9896$ $.9898$ $.9011$ $.9906$ $.9909$ $.9911$ $.9913$ $.3$ $.9893$ $.9896$ $.9898$ $.9011$ $.9906$ $.9909$ $.9911$ $.9913$ $.5$ $.9938$ $.9940$ $.9941$ $.9943$ $.9946$ $.9948$ $.9949$ $.9951$ $.5$ $.9938$ $.9940$ $.9941$ $.9945$ $.9946$ $.9948$ $.9949$ $.9951$ $.7$ $.9965$ $.9966$ $.9967$ $.9967$ $.9977$ $.9977$ $.9972$ $.9973$ $.9974$ $.9975$ $.9976$ $.9977$ $.9977$ $.9978$ $.9985$ $.998$.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.944
.7 $.9554$ $.9564$ $.9573$ $.9582$ $.9591$ $.9599$ $.9608$ $.9616$ $.9625$ $.8$ $.9641$ $.9649$ $.9656$ $.9664$ $.9671$ $.9678$ $.9686$ $.9693$ $.9699$ $.9$ $.9713$ $.9719$ $.9726$ $.9732$ $.9738$ $.9744$ $.9750$ $.9756$ $.9761$ $.0$ $.9772$ $.9778$ $.9783$ $.9788$ $.9793$ $.9798$ $.9803$ $.9808$ $.9812$ $.1$ $.9821$ $.9826$ $.9830$ $.9834$ $.9838$ $.9842$ $.9846$ $.9850$ $.9854$ $.2$ $.9861$ $.9864$ $.9868$ $.9871$ $.9875$ $.9878$ $.9881$ $.9884$ $.9887$ $.3$ $.9893$ $.9896$ $.9898$ $.9901$ $.9904$ $.9906$ $.9909$ $.9911$ $.9913$ $.3$ $.9893$ $.9866$ $.9871$ $.9927$ $.9929$ $.9931$ $.9932$ $.9934$ $.4$ $.9918$ $.9920$ $.9922$ $.9925$ $.9927$ $.9929$ $.9931$ $.9932$ $.9934$ $.6$ $.9953$ $.9955$ $.9956$ $.9957$ $.9959$ $.9960$ $.9961$ $.9962$ $.9963$ $.7$ $.9965$ $.9966$ $.9967$ $.9977$ $.9977$ $.9977$ $.9972$ $.9973$ $.9981$ $.9982$ $.9982$ $.9983$ $.9984$ $.9984$ $.9985$ $.9986$ $.99987$ $.9987$ $.9987$ $.9987$ $.9987$ $.9982$.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.954
.8 .9641 .9649 .9656 .9664 .9671 .9678 .9686 .9693 .9699 .9 .9713 .9719 .9726 .9732 .9738 .9744 .9750 .9756 .9761 .0 .9772 .9778 .9783 .9788 .9793 .9798 .9803 .9808 .9812 .1 .9821 .9826 .9830 .9834 .9838 .9842 .9846 .9850 .9854 .2 .9861 .9864 .9868 .9871 .9875 .9878 .9881 .9884 .9887 .3 .9893 .9896 .9898 .9901 .9904 .9906 .9909 .9911 .9913 .4 .9918 .9920 .9922 .9925 .9927 .9929 .9931 .9932 .9934 .6 .9953 .9956 .9957 .9959 .9966 .9948 .9949 .9951 .7 .9965 .9966 .9967 .9958 .9960 .9971 .9972 .9973 .9974 .9	.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.963
9 $.9713$ $.9719$ $.9726$ $.9732$ $.9738$ $.9744$ $.9750$ $.9756$ $.9761$.0 $.9772$ $.9778$ $.9783$ $.9788$ $.9793$ $.9798$ $.9803$ $.9808$ $.9812$.1 $.9821$ $.9826$ $.9830$ $.9834$ $.9838$ $.9842$ $.9846$ $.9850$ $.9854$.2 $.9861$ $.9864$ $.9868$ $.9871$ $.9875$ $.9878$ $.9881$ $.9884$ $.9887$.3 $.9893$ $.9896$ $.9898$ $.9901$ $.9904$ $.9906$ $.9909$ $.9911$ $.9913$.4 $.9918$ $.9920$ $.9922$ $.9925$ $.9927$ $.9929$ $.9931$ $.9932$ $.9934$.5 $.9938$ $.9940$ $.9941$ $.9943$ $.9945$ $.9946$ $.9948$ $.9949$ $.9951$.5 $.9938$ $.9940$ $.9941$ $.9943$ $.9945$ $.9946$ $.9948$ $.9949$ $.9951$.6 $.9953$ $.9955$ $.9956$ $.9957$ $.9959$ $.9960$ $.9961$ $.9962$ $.9963$.7 $.9965$ $.9966$ $.9967$ $.9968$ $.9969$ $.9971$ $.9972$ $.9973$.9 $.9981$ $.9982$ $.9982$ $.9983$ $.9984$ $.9985$ $.9986$.9987 $.9987$ $.9987$ $.9987$ $.9987$ $.9985$ $.9986$.9990 $.9991$ $.9991$ $.9991$ $.9992$ $.9992$ $.9992$ $.9992$.9993 $.9994$.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	9699	970
.0 $.9772$ $.9778$ $.9783$ $.9788$ $.9793$ $.9798$ $.9803$ $.9808$ $.9812$.1 $.9821$ $.9826$ $.9830$ $.9834$ $.9838$ $.9842$ $.9846$ $.9850$ $.9854$.2 $.9861$ $.9864$ $.9868$ $.9871$ $.9875$ $.9878$ $.9881$ $.9884$ $.9887$.3 $.9893$ $.9896$ $.9898$ $.9901$ $.9904$ $.9906$ $.9909$ $.9911$ $.9913$.4 $.9918$ $.9920$ $.9922$ $.9925$ $.9927$ $.9929$ $.9931$ $.9932$ $.9934$.5 $.9938$ $.9940$ $.9941$ $.9943$ $.9945$ $.9946$ $.9948$ $.9949$ $.9951$.5 $.9938$ $.9940$ $.9941$ $.9943$ $.9945$ $.9946$ $.9948$ $.9949$ $.9951$.6 $.9953$ $.9955$ $.9956$ $.9957$ $.9959$ $.9960$ $.9961$ $.9962$ $.9963$.7 $.9965$ $.9966$ $.9967$ $.9968$ $.9969$ $.9971$ $.9972$ $.9973$.9 $.9981$ $.9982$ $.9982$ $.9983$ $.9984$ $.9984$ $.9985$ $.9986$.9 $.9987$ $.9987$ $.9987$ $.9987$ $.9987$ $.9985$ $.9986$.9990 $.9991$ $.9991$ $.9991$ $.9992$ $.9992$ $.9992$ $.9992$.9993 $.9994$ $.9994$ $.9994$ $.9994$ $.9994$ $.9994$ $.9995$ $.9995$.99	9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	9761	976
.1 .9821 .9826 .9830 .9834 .9838 .9842 .9846 .9850 .9812 .2 .9861 .9864 .9868 .9871 .9875 .9878 .9881 .9884 .9887 .3 .9893 .9896 .9898 .9901 .9904 .9906 .9909 .9911 .9913 .4 .9918 .9920 .9922 .9925 .9927 .9929 .9931 .9932 .9934 .5 .9938 .9940 .9941 .9943 .9945 .9946 .9948 .9949 .9951 .5 .9938 .9955 .9956 .9957 .9959 .9960 .9961 .9962 .9953 .9955 .9956 .9957 .9959 .9960 .9961 .9962 .9963 .9965 .9966 .9967 .9968 .9969 .9971 .9972 .9973 .9965 .9966 .9967 .9967 .9978 .9979 .9979 .9973 .9974 .9975 .9976 .9977 .9978	.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	9808	0812	081
2 .9861 .9864 .9868 .9871 .9875 .9878 .9881 .9884 .9887 3 .9893 .9896 .9898 .9901 .9904 .9906 .9909 .9911 .9913 4 .9918 .9920 .9922 .9925 .9927 .9929 .9931 .9932 .9934 5 .9938 .9940 .9941 .9943 .9945 .9946 .9948 .9949 .9951 5 .9938 .9955 .9956 .9957 .9959 .9960 .9961 .9962 .9963 6 .9953 .9956 .9957 .9959 .9960 .9961 .9962 .9963 7 .9965 .9966 .9967 .9977 .9978 .9972 .9973 9974 .9975 .9976 .9977 .9978 .9979 .9979 .9980 9.9981 .9982 .9983 .9984 .9984 .9985 .9986 .9986 .9989 .9989 .9989 .9989 .9986 .9986 .9986 .	1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	9850	0854	.901
.3 .9893 .9896 .9898 .9901 .9904 .9906 .9909 .9911 .9913 .4 .9918 .9920 .9922 .9925 .9927 .9929 .9931 .9932 .9934 .5 .9938 .9940 .9941 .9943 .9945 .9946 .9948 .9949 .9951 .6 .9953 .9955 .9956 .9957 .9959 .9960 .9961 .9962 .9963 .7 .9965 .9966 .9967 .9968 .9970 .9971 .9972 .9973 .9965 .9966 .9967 .9977 .9978 .9979 .9972 .9973 .9981 .9982 .9982 .9983 .9984 .9984 .9985 .9980 .9981 .9982 .9987 .9987 .9988 .9984 .9985 .9986 .9987 .9987 .9987 .9988 .9988 .9989 .9989 .9990 .9990 .9991 .9991 .9991 .9992 .9992 .9992 .9993	2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	9884	.9094	.90.
4 .9918 .9920 .9922 .9925 .9927 .9929 .9931 .9932 .9934 5 .9938 .9940 .9941 .9943 .9945 .9946 .9948 .9949 .9951 6 .9953 .9955 .9956 .9957 .9959 .9960 .9961 .9962 .9963 7 .9965 .9966 .9957 .9959 .9960 .9971 .9972 .9973 8 .9974 .9975 .9976 .9977 .9978 .9979 .9979 .9973 9 .9981 .9982 .9983 .9984 .9984 .9985 .9980 9.9981 .9982 .9987 .9988 .9984 .9985 .9986 .9986 9.9987 .9987 .9987 .9988 .9988 .9989 .9989 .9986 .9986 .9986 .9986 .9986 .9989 .9986 .9986 .9989 .9990 .9991 .9991 .9991 .9992 .9992 .9992 .9992 .9993 .9993 .9994	3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	0011	.900/	.90
5 .9938 .9940 .9941 .9943 .9945 .9946 .9948 .9949 .9951 6 .9953 .9955 .9956 .9957 .9959 .9960 .9961 .9962 .9963 7 .9965 .9966 .9967 .9968 .9969 .9970 .9971 .9972 .9973 8 .9974 .9975 .9976 .9977 .9978 .9979 .9972 .9973 9 .9981 .9982 .9982 .9983 .9984 .9984 .9985 .9980 0 .9987 .9987 .9987 .9988 .9988 .9989 .9985 .9986 0 .9987 .9987 .9987 .9988 .9988 .9989 .9989 .9986 0 .9987 .9987 .9984 .9992 .9992 .9992 .9992 .9992 .9993 0 .9993 .9994 .9994 .9994 .9994 .9994 .9995 .9995 .9995 .9996 .9996 .9996 .9996 .9996 <td>4</td> <td>.9918</td> <td>.9920</td> <td>.9922</td> <td>.9925</td> <td>.9927</td> <td>.9929</td> <td>.9931</td> <td>.9911</td> <td>.9913</td> <td>.99.</td>	4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9911	.9913	.99.
6 .9953 .9955 .9956 .9957 .9959 .9960 .9961 .9962 .9963 7 .9965 .9966 .9967 .9968 .9969 .9970 .9971 .9972 .9973 8 .9974 .9975 .9976 .9977 .9977 .9978 .9979 .9979 .9973 9 .9981 .9982 .9983 .9984 .9984 .9985 .9986 .9986 0 .9987 .9987 .9987 .9988 .9988 .9989 .9985 .9986 0 .9981 .9991 .9991 .9992 .9992 .9992 .9990 2 .9993 .9994 .9994 .9994 .9994 .9992 .9992 .9992 .9993 2 .9993 .9995 .9996 .9996 .9996 .9996 .9995 .9995 .9995 .9995 .9995 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .99	5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9932	.9934	.99.
7 .9965 .9966 .9967 .9968 .9969 .9970 .9971 .9972 .9973 8 .9974 .9975 .9976 .9977 .9977 .9978 .9979 .9972 .9973 9 .9981 .9982 .9982 .9983 .9984 .9985 .9985 .9986 0 .9987 .9987 .9987 .9988 .9988 .9989 .9985 .9986 0 .9990 .9991 .9991 .9992 .9992 .9992 .9992 .9992 .9993 2 .9993 .9995 .9996 .9996 .9996 .9995 .9995 .9996 .9996 .9995 .9995 .9995 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9995 .9995 .9997 .9997 .9995 .9997 .9997 .9995 .9997 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996	6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9949	.9951	.995
8 .9974 .9975 .9976 .9977 .9977 .9978 .9979 .9972 .9973 9 .9981 .9982 .9982 .9983 .9984 .9984 .9979 .9979 .9980 0 .9987 .9987 .9987 .9988 .9984 .9985 .9985 .9986 0 .9990 .9991 .9991 .9992 .9992 .9989 .9990 2 .9993 .9994 .9994 .9994 .9994 .9994 .9992 .9992 .9992 .9993 2 .9995 .9995 .9996 .9996 .9996 .9996 .9995 .9995 .9995 .9996 .9996 .9996 .9996 .9996 .9995 .9995 .9997 .9997 .9997 .9997 .9997 .9997 .9995 .9997 .9996	7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9962	.9963	.990
9 .9981 .9982 .9982 .9983 .9984 .9984 .9979 .9979 .9980 0 .9987 .9987 .9987 .9988 .9988 .9989 .9985 .9985 .9986 1 .9990 .9991 .9991 .9991 .9992 .9992 .9989 .9990 2 .9993 .9994 .9994 .9994 .9994 .9994 .9992 .9992 .9992 .9993 3 .9995 .9995 .9996 .9996 .9996 .9996 .9995 .9995 .9995 .9996 .9996 .9996 .9996 .9996 .9995 .9995 .9995 .9997<	3	.9974	.9975	.9976	.9977	.9977	.9978	.9970	.9972	.9973	.99
0 .9987 .9987 .9987 .9988 .9988 .9989 .9989 .9985 .9986 1 .9990 .9991 .9991 .9991 .9992 .9992 .9989 .9989 .9990 2 .9993 .9993 .9994 .9994 .9994 .9994 .9992 .9992 .9992 .9993 3 .9995 .9995 .9996 .9996 .9996 .9996 .9995 .9995 4 .9997<)	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9979	.9980	.998
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3 .9995 .9995 .9995 .9996 .9996 .9996 .9996 .9996 .9995 .9995 4 .9997 .9997 .9997 .9997 .9997 .9997 .9996 .9996 .9996		.9993	.9993	.9994	.9994	.9994	.9994	.9904	.9992	.9993	.99
.9997 .9997 .9997 .9997 .9997 .9997 .9996 .9996		.9995	.9995	.9995	.9996	.9996	.9996	9006	.9995	.9995	.99
.999/ 0007 .999/ 9996	-	.9997	.9997	.9997	.9997	.9997	.9997	0007	.9996	.9996	99



Department of Mathematics

3/12/2025

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Friday's draft problem

To be presented by Friday's draftee.

Variation of Ex. 4d (with more accurate numbers)

An expert witness in a paternity suit testifies that the length (in days) of human gestation is approximately 280 days with a standard deviation of 14 days. The defendant in the suit (a long haul trucker) is able to prove that he was not in the mother's time-zone for a period of 40 days that began 300 days before the child's birth.

If the defendant were the father, what is the probability that the mother had the kind of exceptionally long or short gestation period implied by the evidence?



