MA 562 - Introduction to Differential Geometry and Topology

Fall 2024

Course Overview

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Important Resources

Syllabus (grading, policies, link to textbook)
https://www.math.purdue.edu/~esampert/562/

Course calendar (reading, homework assignments, lecture notes, dead links will get filled in as semester proceeds)
https://www.math.purdue.edu/~esampert/562/cal

Brightspace (announcements, homework submission, gradebook)
https://purdue.brightspace.com/d2l/home/1102972

MediaSpace Channel (delayed live stream, recordings)
https://mediaspace.itap.purdue.edu/channel/Fall+2024+-+MA562+-+Samperton+(8:30)/351337722

Course Content - Five Parts

First four parts cover Chapters 1-17 of Lee (sans 13). 25 minute in-class quiz after each of the first four parts. Dates TBD soon. HW every week. Roughly 2 days/chapter.

1. Smooth manifolds and smooth maps
   - Topological manifolds, examples, smooth atlas, tangent vectors, derivatives. Chapters 1-3.
2. Local structure of smooth maps and global embedding theorems
   - Immersions, submersion, Sard's theorem, Whitney embedding theorem. Chapters 4-6.
3. Integration I - Vector Fields
   - Lie groups and Lie algebras, integral curves. Chapters 7-9
4. Integration II - Differential Forms
   - Tangent and cotangent bundles, orientations, Stoke's theorem, de Rham cohomology. Chapters 10-16 (skip Ch. 13)
5. Riemannian Manifolds (time permitting)
   - Very brief. Curves and surfaces, Gauss-Bonnet

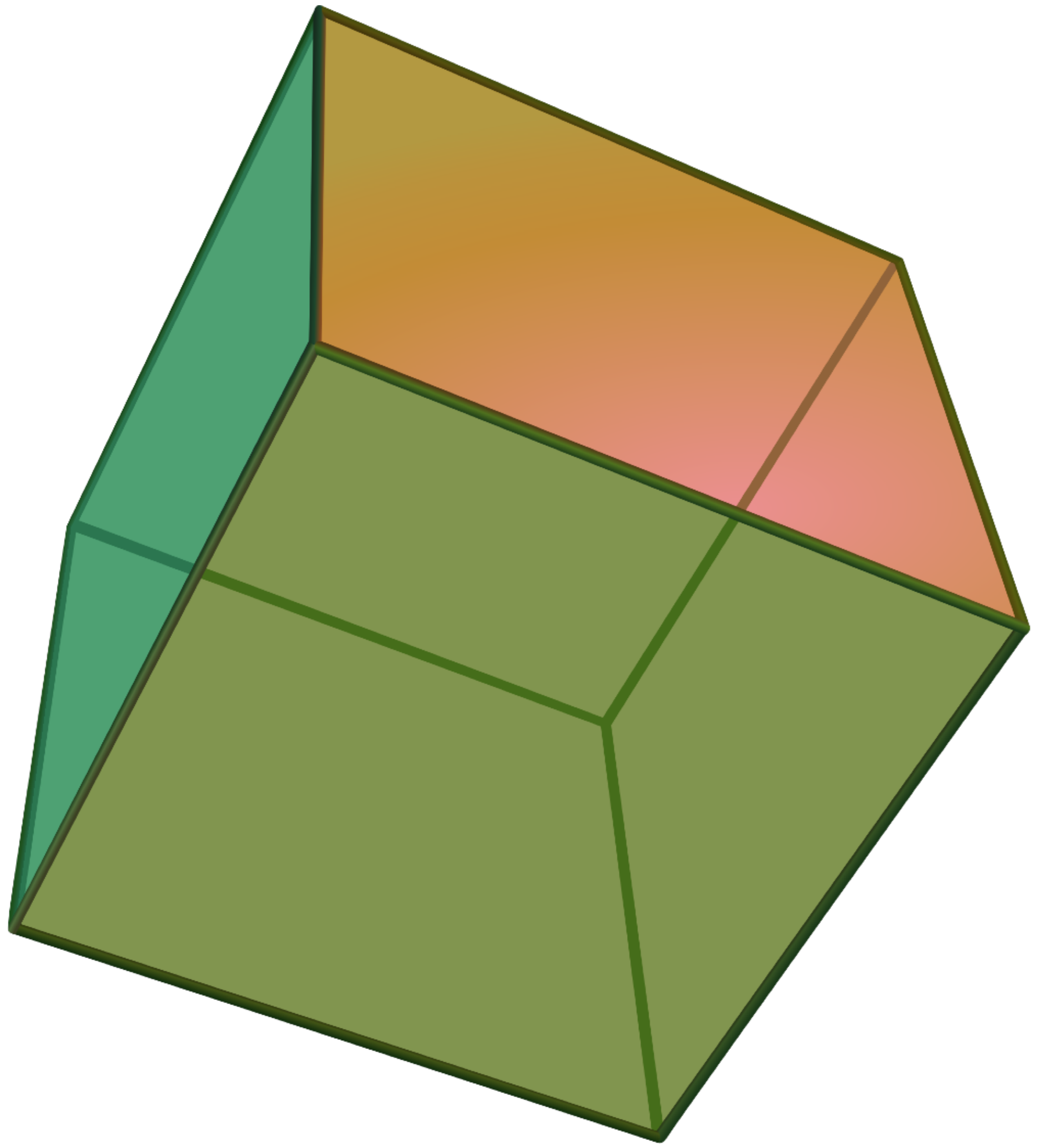
Comments about prerequisites & the textbook

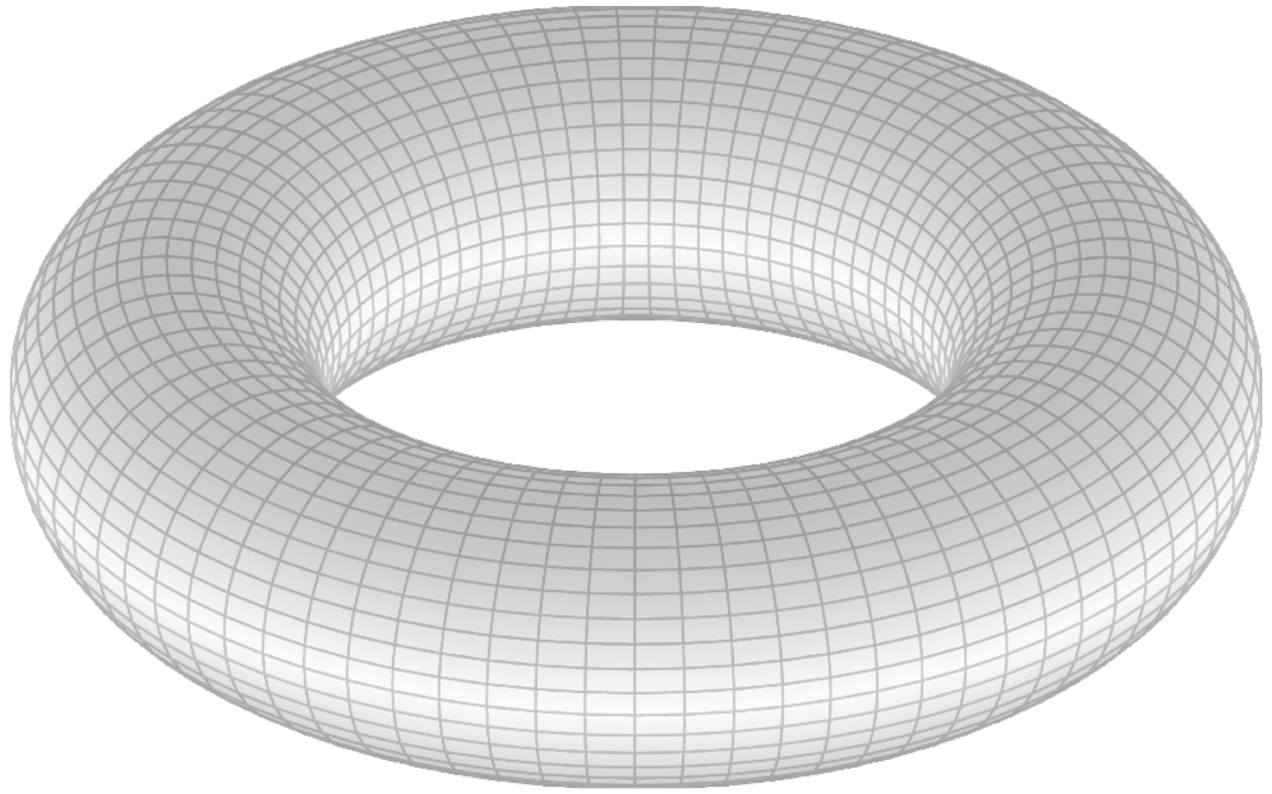
Point-set topology is not strictly a pre-requisite for this course, but we will be using some basic definitions from it. Read Appendix A if you need to learn these.

On the other hand, you can ignore things about the “fundamental group” or “covering spaces” unless I say otherwise.

What's a manifold?

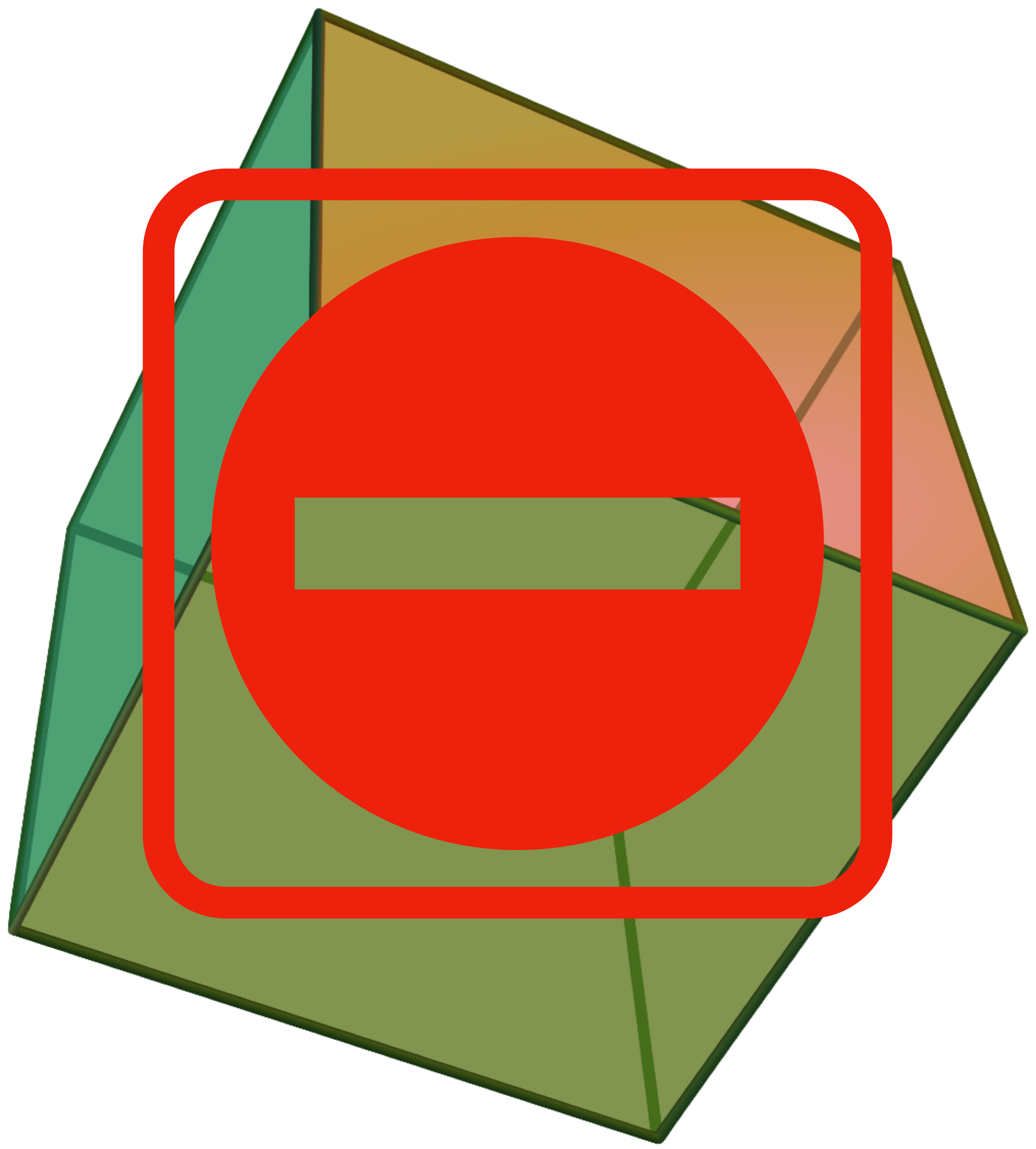
A *topological manifold* of dimension n is any topological space that locally looks like $\mathbb{R}^n$.

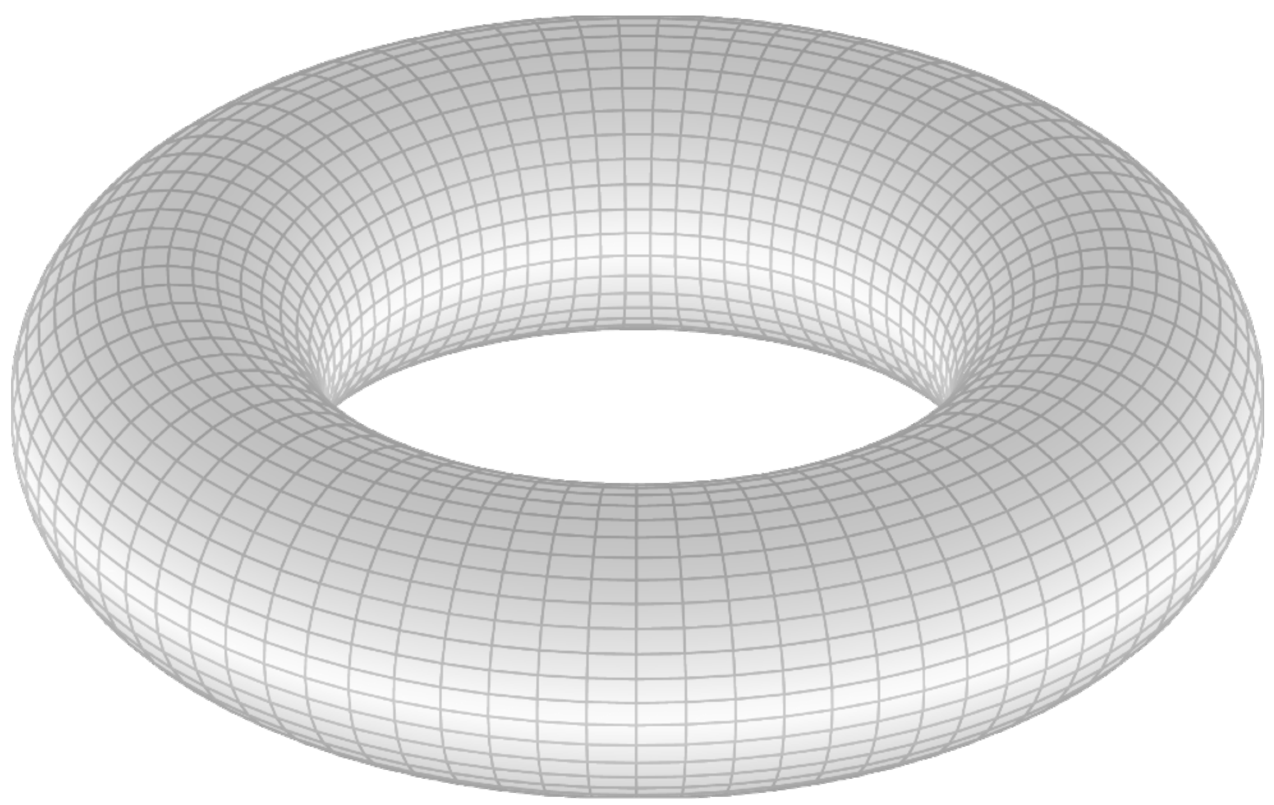


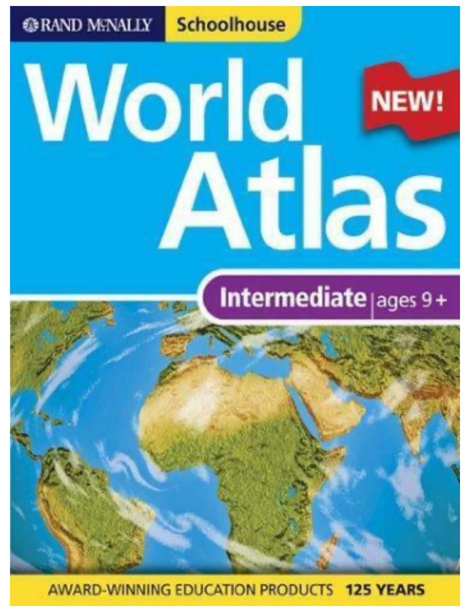


What’s a *smooth* manifold?

A *smooth manifold* is a topological manifold equipped with data that allows us to “do calculus on it.” Called an *atlas of charts*.

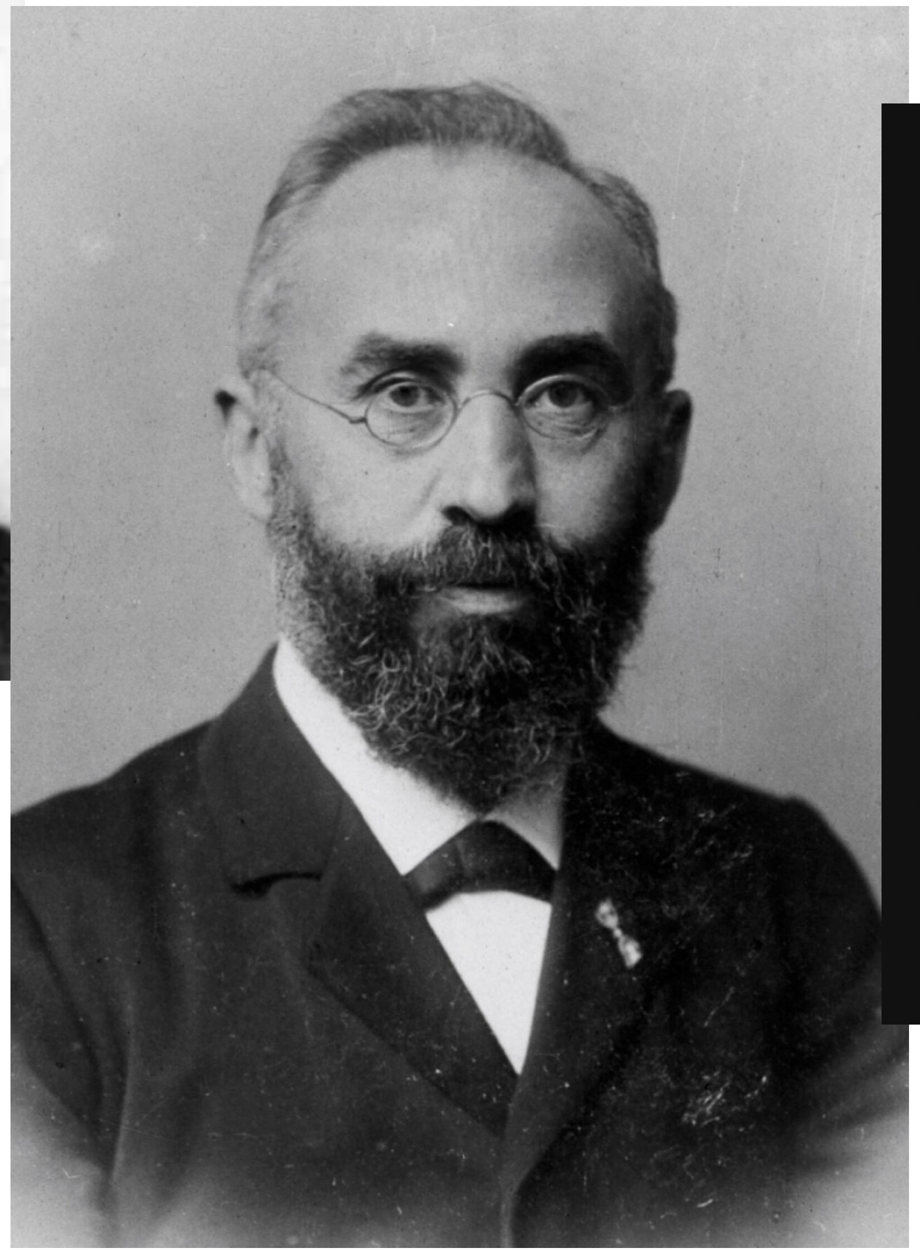






Why care?

- Both smooth and topological manifolds are interesting to study on their own terms ("geometric topology").
- Smooth manifolds are an especially useful idea because they support the necessary structures to do "very general" analytic geometry ("Riemannian geometry").
- This class will mostly focus on smooth manifolds (introductory "differential topology").



Why care?

Numerous applications: classical mechanics, cartography, general relativity, differential equations (e.g. free boundary problems), quantum field theory, …

“Philosophically:” manifolds are the archetypal examples of mathematical objects with interesting “global” structure that is built by pasting together “simple” local pieces. This is a useful idea that generalizes to other types of math.

Intrinsic beauty