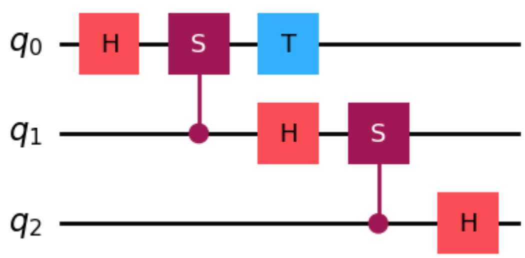
CS 593/MA 595 - Intro to Quantum Computation
Midterm Exam 1 PRACTICE

Note: on the actual exam, we'll remind you what the matrix forms of $H, CNOT$, Toffoli, S, T, R_x, R_y, and R_z are.

1. Let's do some calculations.

   (a) Consider the following circuit C:

   Express $C|101\rangle$ explicitly as a column vector with respect to the computational basis. (Hint: write $\omega = e^{2\pi i/8}$.)

   (b) Let

   $$M = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$

   be an observable on 2 qubits and consider an arbitrary (normalized) state

   $$|\psi\rangle = z_00|00\rangle + z_{01}|01\rangle + z_{10}|10\rangle + z_{11}|11\rangle.$$

   Compute (in terms of the z_{jk}'s) the probability distribution of outcomes if we measure M on $|\psi\rangle$.

2. Let MAJ be the 3-ary Boolean function that takes the majority vote of 3 bits.

xyz	$MAJ(x, y, z)$
000	0
001	0
010	0
011	1
100	0
101	1
110	1
111	1

Find a quantum circuit C (using only single qubit gates, CNOT or Toffoli gates) so that $C|xyzw\rangle = |xyz\rangle \otimes |MAJ(x,y,z) \oplus w\rangle$.

3. Suppose Alice and Bob share a Bell pair, and suppose Alice has another two qubits in some state $|\psi\rangle \in \left(\mathbb{C}^2\right)^{\otimes 2}$. Alice and Bob then use their Bell pair to teleport one of her two qubits to Bob. Show that the resulting state on Bob's qubit is entangled with Alice's remaining qubit if and only if the state $|\psi\rangle$ was entangled. (Recall: a state $|\psi\rangle \in \left(\mathbb{C}^2\right)^{\otimes 2}$ is called *un-entangled* if and only if it can be written as $|\psi\rangle = |\phi_1\rangle \otimes |\phi_2\rangle$ for some states $|\phi_1\rangle, |\phi_2\rangle \in \mathbb{C}^2$.)