CS 593/MA 595 - Intro to Quantum Computation
Theoretical Homework 3

Due Wednesday, September 17 at 11:59PM (upload to Brightspace)

Recommended exercises from Mike and Ike (not to be turned in): 2.65, 2.67, 2.68, 2.69.

1. (Distinguishability of quantum states) Recall that, in the class, we defined

$$d(|\psi\rangle, |\varphi\rangle) = \max_{\mathrm{spec}(M)\subseteq\{-1,+1\}} \frac{1}{2} \left| \langle\psi| M |\psi\rangle - \langle\varphi| M |\varphi\rangle \right|.$$

Prove that

$$d(|\psi\rangle, |\varphi\rangle) = \sqrt{1 - |\langle\psi|\varphi\rangle|^2}.$$

Hint: Consider the 2-d vector space spanned by $|\psi\rangle$ and $|\varphi\rangle$. What does a measurement M restricted in this space look like? Then, parameterize the angles.

Notice that unitary does not change the angles between transformed states. Together with the deferred measurement principle, this result implies that: for two non-orthogonal quantum states, there is no way to perfectly distinguish them.

2. (Quantum upper bounds of CHSH). The following is a formal definition of the CHSH quantity S: Consider the scenario where Charlie distributes parts of a quantum state $|\lambda\rangle$ to Alice and Bob. Alice has a secret bit x and can perform a binary measurement M_x^A (recall the definition of binary: $\mathrm{spec}(M_x^A) = \{-1, +1\}$) locally to $|\lambda\rangle$, and so does Bob (with a secret bit y, and a binary measurement M_y^B). Since the measurements are performed locally, there is $[M_x^A, M_y^B] = 0$ for all x, y. Then, we can calculate the average product

$$\langle ab\rangle_{xy} = \langle\lambda| M_x^A M_y^B |\lambda\rangle,$$

and define quantity

$$S = \langle ab\rangle_{00} + \langle ab\rangle_{01} + \langle ab\rangle_{10} - \langle ab\rangle_{11}.$$

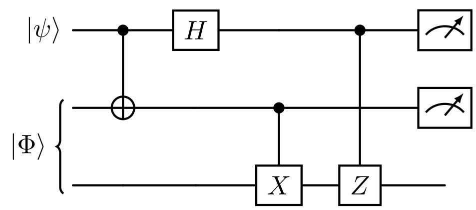
Your task is to prove that

$$S \leq 2\sqrt{2}.$$

Hint: First prove that, for any Hermitian M and quantum state $|\psi\rangle$, there is $(\langle\psi| M |\psi\rangle)^2 \leq \langle\psi| M^2 |\psi\rangle$. Then consider S^2.

3. Evaluate the outputs of the following quantum circuits. List the outcomes of the measurements with their probability, and the states for qubits that are not measured. The measurements in this problem are Z basis measurements (computational basis).

   (a) In the following circuit, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, and $|\Phi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.



(b) In the following circuit, $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,, |\phi\rangle = \gamma\,|0\rangle + \delta\,|1\rangle\,.$