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ORIGINAL ARTICLE

Epidemiological Models with Non-Exponentially Distributed Disease Stages and Applications to Disease Control

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Abstract *SEIR* epidemiological models with the inclusion of quarantine and isolation are used to study the control and intervention of infectious diseases. A simple ordinary differential equation (ODE) model that assumes exponential distribution for the latent and infectious stages is shown to be inadequate for assessing disease control strategies. By assuming arbitrarily distributed disease stages, a general integral equation model is developed, of which the simple ODE model is a special case. Analysis of the general model shows that the qualitative disease dynamics are determined by the reproductive number $\mathcal{R}_c$, which is a function of control measures. The integral equation model is shown to reduce to an ODE model when the disease stages are assumed to have a gamma distribution, which is more realistic than the exponential distribution. Outcomes of these models are compared regarding the effectiveness of various intervention policies. Numerical simulations suggest that models that assume exponential and non-exponential stage distribution can produce inconsistent predictions.

Keywords Epidemiological model · Distributed disease stage · Integral equation · Disease control strategies

1. Introduction

The mathematical theory of infectious diseases pioneered by Ross, MacDonald, Kermack, McKendrick and others has played a major role in the study of the control and prevention of infectious diseases (see, for example, Ross, 1911; Kermack and McKendrick, 1927). More recently, mathematical models have been used to investigate how to more effectively control SARS via various disease control measures including vaccination, quarantine, and isolation (see, for

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