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ORIGINAL ARTICLE

Transmission Dynamics of an Influenza Model with Vaccination and Antiviral Treatment

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Abstract Vaccination and antiviral treatment are two important prevention and control measures for the spread of influenza. However, the benefit of antiviral use can be compromised if drug-resistant strains arise. In this paper, we develop a mathematical model to explore the impact of vaccination and antiviral treatment on the transmission dynamics of influenza. The model includes both drug-sensitive and resistant strains. Analytical results of the model show that the quantities $\mathcal{R}_{SC}$ and $\mathcal{R}_{RC}$, which represent the control reproduction numbers of the sensitive and resistant strains, respectively, provide threshold conditions that determine the competitive outcomes of the two strains. These threshold conditions can be used to gain important insights into the effect of vaccination and treatment on the prevention and control of influenza. Numerical simulations are also conducted to confirm and extend the analytic results. The findings imply that higher levels of treatment may lead to an increase of epidemic size, and the extent to which this occurs depends on other factors such as the rates of vaccination and resistance development. This suggests that antiviral treatment should be implemented appropriately.

Keywords Influenza · Antiviral treatment · Vaccination · Stability · Drug-resistant strains

1. Introduction

Influenza (the flu) is a contagious respiratory illness caused by influenza viruses, which are certain RNA viruses of the *Orthomyxoviridae* family (Lamb, 1989; Earn et al., 2002). In humans, common symptoms of influenza infection are fever, sore throat, muscle pains, severe headache, coughing, and weakness and fatigue. Although it is sometimes confused with the common cold, influenza is a much more severe disease. It has historically been

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