

# Discrete Epidemic Models with Arbitrary Stage Distributions and Applications to Disease Control

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**Abstract** W.O. Kermack and A.G. McKendrick introduced in their fundamental paper, *A Contribution to the Mathematical Theory of Epidemics*, published in 1927, a deterministic model that captured the qualitative dynamic behavior of single infectious disease outbreaks. A Kermack–McKendrick discrete-time general framework, motivated by the emergence of a multitude of models used to forecast the dynamics of epidemics, is introduced in this manuscript. Results that allow us to measure quantitatively the role of classical and general distributions on disease dynamics are presented. The case of the geometric distribution is used to evaluate the impact of waiting-time distributions on epidemiological processes or public health interventions. In short, the geometric distribution is used to set up the baseline or null epidemiological model used to test the relevance of realistic stage-period distribution on the dynamics of single epidemic outbreaks. A final size relationship involving the control reproduction number, a function of transmission parameters and the means of distributions used to model disease or intervention control measures, is computed. Model results and simulations highlight the inconsistencies in forecasting that emerge from the use of specific parametric distributions. Examples, using the geometric, Poisson and binomial distributions, are used to highlight the impact of the choices made in quantifying the risk posed by single outbreaks and the relative importance of various control measures.

**Keywords** Discrete-time model · Epidemics · Control reproduction number · Final epidemic size · Arbitrarily distributed disease durations

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