DISCRETE AND CONTINUOUS
DYNAMICAL SYSTEMS SERIES B
Volume **21**, Number **2**, March **2016**

doi:10.3934/dcdsb.2016.21.399

pp. **399–415**

COMPUTATION OF $\mathcal{R}$ IN AGE-STRUCTURED EPIDEMIOLOGICAL MODELS WITH MATERNAL AND TEMPORARY IMMUNITY

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Abstract. For infectious diseases such as pertussis, susceptibility is determined by immunity, which is chronological age-dependent. We consider an age-structured epidemiological model that accounts for both passively acquired maternal antibodies that decay and active immunity that wanes, permitting re-infection. The model is a 6-dimensional system of partial differential equations (PDE). By assuming constant rates within each age-group, the PDE system can be reduced to an ordinary differential equation (ODE) system with aging from one age-group to the next. We derive formulae for the effective reproduction number $\mathcal{R}$ and provide their biological interpretation in some special cases. We show that the disease-free equilibrium is stable when $\mathcal{R} < 1$ and unstable if $\mathcal{R} > 1$.

1. **Introduction.** When modeling infectious diseases such as pertussis, age-dependent immunity and susceptibility to disease are important to consider. One of the main reasons is that infants may receive maternal antibodies that do not confer permanent immunity, permitting re-infection after immunity wanes. In [3], an age-structured model is used to estimate the age-dependent rates of infection for pertussis using data from serological surveys in Sweden. This is done by fitting the data to age-dependent probabilities of being infected once or twice. One approach used to

2010 *Mathematics Subject Classification.* Primary: 92B05, 92D30; Secondary: 92D25.

Key words and phrases. Age-structured epidemiological model, reproduction numbers, partial immunity, multiple infections.

This research is supported in part by the NSF grant DMS-1022758. Zhipeng Qiu is supported by the NSFC grant 11271190.