Journal of Theoretical Biology 356 (2014) 123–132

Contents lists available at ScienceDirect

Journal of Theoretical Biology

journal homepage: www.elsevier.com/locate/yjtbi

Modeling rates of infection with transient maternal antibodies and waning active immunity: Application to *Bordetella pertussis* in Sweden

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HIGHLIGHTS

- We modeled age-specific forces of infection when immunity wanes in Sweden
- We used a cross-sectional survey of anti-PT after a 17-year vaccination hiatus
- Using Finnish contact rates, we estimated probabilities of infection on contact
- We also estimated the infection rates, $\mathfrak{R}_0$, and age-specific contributions
- Our $\mathfrak{R}_0$ approximates the ratio of longevity and age at first infection

ARTICLE INFO

Article history:
Received 29 April 2013
Received in revised form
15 April 2014
Accepted 15 April 2014
Available online 23 April 2014

Keywords:
Catalytic modeling
Force of infection
Antibodies to pertussis toxin
Waning immunity
Reproduction number

ABSTRACT

Serological surveys provide reliable information from which to calculate forces (instantaneous rates) of infection, but waning immunity and clinical consequences that depend on residual immunity complicate interpretation of results. We devised a means of calculating these rates that accounts for passively acquired maternal antibodies that decay or active immunity that wanes, permitting re-infection. We applied our method to pertussis (whooping cough) in Sweden, where vaccination was discontinued from 1979 to 1995. A national cross-sectional serosurvey of antibodies to pertussis toxin, which peak soon after infection and then decay, was conducted shortly after vaccination resumed. Together with age-specific contact rates in Finland, contemporary forces of infection enable us to evaluate the recent assertion that the probability of infection upon contact is age-independent. We find elevated probabilities among children, adolescents and young adults, whose contacts may be more intimate than others. Products of contact rates and probabilities of infection permit transmission modeling and estimation of the intrinsic reproduction number. In contrast to another recent estimate, ours approximates the ratio of life expectancy and age at first infection. Our framework is sufficiently general to accommodate more realistic sojourn distributions and additional lifetime infections.

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1. Introduction

Ever since Muench (1959) pioneered catalytic modeling, epidemiologists have used functions that increase at decreasing rates over the unit interval to estimate forces of infection among susceptible persons from cross-sectional serological surveys. This approach was elaborated by others (Grenfell and Anderson, 1985; Griffiths, 1974), as well as Farrington (1990), who subsequently concluded that, however unrealistic biologically, the equilibrium assumption underlying this approach was inconsequential (Whitaker and Farrington, 2004). But these methods ignore passively acquired maternal antibodies

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http://dx.doi.org/10.1016/j.jtbi.2014.04.020
0022-5193/Published by Elsevier Ltd.