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# DYNAMICS OF A PLANT-HERBIVORE MODEL WITH TOXIN-INDUCED FUNCTIONAL RESPONSE

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*Dedicated to Professor Horst R. Thieme on the Occasion of his 60th Birthday*

Abstract. Traditional functional responses for plant-herbivore interactions do not take into account explicitly the effect of plant toxin. However, considerable evidence suggests that toxins set upper limits on food intake for many species of herbivorous vertebrates. In this paper, a mathematical model for plant-herbivore interactions mediated by toxin-determined functional response is studied. The model consists of three ordinary differential equations describing one herbivore population and two plant species with different toxicity levels. The effect of plant toxicity on herbivore's intake rate is incorporated explicitly in the model by assuming an increased handling time. The dynamical behaviors of the model are analyzed and the results are used to examine the influence of toxin-determined intake in the community composition of plant species. The bifurcation analysis presented in this paper suggests that the toxin-mediated functional response may have dramatic effects on plant-herbivore interactions.

1. **Introduction.** Over the past two decades, ecologists have focused intensively on chemically mediated plant-herbivore interactions [5, 10, 13, 18], and suggested that plant toxins play an important role in regulating herbivore's consumption of the plant [11, 12, 17, 18, 19]. Specific examples and more detailed discussions about the importance of plant toxins as determinants of herbivore functional response can be found in [16]. Although the impact of toxins on herbivores' diet has been emphasized in a great deal of research, it is frequently ignored in plant-herbivore models (e.g., [1, 8, 9, 15]). In [6], we constructed a toxin-determined functional response model (referred to as TDRFM) that explicitly incorporates the effect of

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