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Variability order of the latent and the infectious periods in a deterministic SEIR epidemic model and evaluation of control effectiveness

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ABSTRACT

We use distribution theory and ordering of non-negative random variables to study the Susceptible-Exposed-Infectious-Removed (SEIR) model with two control measures, quarantine and isolation, to reduce the spread of an infectious disease. We identify that the probability distributions of the latent period and the infectious period are primary features of the SEIR model to formulate the epidemic threshold and to evaluate the effectiveness of the intervention measures. If the primary features are changed, the conclusions will be altered in an importantly different way. For the latent and infectious periods with known mean values, it is the dilation, a generalization of variance, of their distributions that ranks the effectiveness of these control measures. We further propose ways to set quarantine and isolation targets to reduce the controlled reproduction number below the threshold using observed initial growth rate from outbreak data. If both quarantine and isolation are 100% effective, one can directly use the observed growth rate for setting control targets. If they are not 100% effective, some further knowledge of the distributions is required.

1. Introduction

While applying a mathematical model to a real life problem, it is important to distinguish the primary features from the secondary features of the model. If a primary feature is changed, the research questions of interest are either changed or at least formulated in an importantly different way. If a secondary feature is changed, the research questions are essentially unaltered. Mis-formulation of the primary features leads to the wrong question being addressed [1].

The research questions posed concern the formulation of the epidemic threshold and the evaluation of two public health intervention measures to reduce the spread of an infectious disease. The features to be discussed are the characteristics of probability distributions of two important time durations along the disease history: the *latent period*, during which the infected individual is not yet infectious, and the *infectious period*, during which the infected individual is able to infect other susceptible individuals through contacts. Some public health control measures aimed to reduce the transmission apply to infected individuals during either of these periods. We illustrate our findings for the following two control measures, both by removing exposed and infected individuals from contacting other susceptible individuals in the population. The difference is between:

1. removing infected individuals during their latent period; and
2. removing infected individuals during their infections period.

For the second action, we use the term *isolation*, which is in agreement with that used in most public health literature.

For the first action, without a better terminology, we use the term *quarantine*. This usage is in agreement with that used in some public health literature, but is quite different in meaning from many other literature and in the media, such as putting an entire school, a community or a plane load of passengers under 'quarantine'. For the rest of this paper, whenever the word quarantine appears, we restrict to our definition, as illustrated in Fig. 1.

Section 2 introduces notations, the model structure and assumptions (features). It also includes a brief review of studies related to the topic of this paper in the literature.

Section 3 formulates the controlled reproduction number, an important epidemic threshold parameter, through ordering of the probability distributions of the latent and the infectious periods. It uses the Laplace transform order to formulate the controlled reproduction number and the relative effectiveness of the control measures. It uses concave order, dilation order and comparing variances to provide intuitive interpretations of the theory. These are stochastic orders on variability on non-negative random variables.

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