Math. Model. Nat. Phenom.
Vol. 3, No. 7, 2008, pp. 180-193

Analysis of a Model with Multiple Infectious Stages and Arbitrarily Distributed Stage Durations

Y. Yang[a], D. Xu[b], and Z. Feng[a1]

[a] Department of Mathematics, Purdue University, West Lafayette, IN 47907, USA
[b] Department of Mathematics, Southern Illinois University, Carbondale, IL 62901, USA

Abstract. Infectious diseases may have multiple infectious stages with very different epidemiological attributes, including infectivity and disease progression. These stages are often assumed to have exponentially distributed durations in epidemiological models. However, models that use the exponential distribution assumption (EDA) may generate biased and even misleading results in some cases. This discrepancy is particularly damaging if the models are employed to assist policy-makers in disease control and interventions. This paper studies a mathematical model that includes multiple infectious stages and general distributions for the stage durations (with the exponential distribution as a special case). Formulas for the control reproductive number, $\mathcal{R}_c$, and the basic reproductive number, $\mathcal{R}_0$, are derived, which can be conveniently applied to models in which specific stage distributions are assumed. It is also shown that the disease dynamics are determined by the reproductive numbers.

Key words: epidemiological model, disease stage distribution, disease control
AMS subject classification: 37N25, 92D30

1 Introduction

One of the most common assumptions used in epidemiological models is that the disease stages are exponentially distributed. That is, the probability that an individual will remain in a disease stage s time units after entering the stage is described by an exponential function of the form $e^{-\theta s}$, where $\theta > 0$ is a constant with $1/\theta$ representing the mean period of staying

[1]Corresponding author. Email: zfeng@math.purdue.edu