Mathematics 174

Division 2, Section 1

Test 1

February 19, 1988

SHOW YOUR WORK

SKETCH means DRAW CAREFULLY WITHOUT A DRAFTING KIT.

(1) Express $(\mathbf{a}+\mathbf{b})\times(\mathbf{a}-\mathbf{b})$ as a scalar multiple of $\mathbf{a}\times\mathbf{b}$.

(2) Recall that if $\mathbf{r}(t)=(x(t),y(t),z(t))$ is a space curve from $t=0$ to $t=1$, then the arc length $s(t)$ is given by

$$s(t)=\int_0^t \|\mathbf{r}'(\tau)\|\,d\tau.$$

Let

$$\mathbf{r}(t)=t\mathbf{i}+2e^t\mathbf{j}+e^{2t}\mathbf{k}.$$

(a) Find $\mathbf{r}'(0)$. Find an equation for the line tangent to the curve at $(0,2,1)$.

(b) Find an equation for the plane normal to the curve at $(0,2,1)$.

(c) Find $\dfrac{d\mathbf{r}}{ds}(0)$. Find the unit tangent vector $\mathbf{T}(t)$.

(d) Find the curvature of the curve at $(0,2,1)$.

(e) What is the length of the curve between $(0,2,1)$ and $(1,2e,e^2)$?

(3) A planet travels around the sun in an elliptical orbit of the form

$$\mathbf{r}(t)=2\cos t\,\mathbf{i}+1.5\sin t\,\mathbf{j}.$$

Find the velocity $\mathbf{v}(t)$ and the acceleration $\mathbf{a}(t)$. What is the maximum magnitude of the acceleration?

(4) Sketch in the x–y plane the level curve $f(x,y)=0$ when

$$f(x,y)=\ln(x^2y^2).$$

(5) Sketch the surface

$$x^2-y^2+z^2+1=0.$$