MA 174: Multivariable Calculus

EXAM I

Feb. 14, 2007

NAME ______________________ INSTRUCTOR ______________________

NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

Points awarded

1. (5 pts) ________	**7. (5 pts)** ________
2. (5 pts) ________	**8. (5 pts)** ________
3. (5 pts) ________	**9. (5 pts)** ________
4. (5 pts) ________	**10. (5 pts)** ________
5. (5 pts) ________	**11. (5 pts)** ________
6. (5 pts) ________	

Total Points: ________/55

1. The plane S passes through the point $P(1,2,3)$ and contains the line $x = 3t$, $y = 1+t$, and $z = 2-t$. Which of the following is an equation for S?

A. $x + 2y + z = 0$

B. $x - 2y + z = 0$

C. $x - 2y + z = 5$

D. $x + 2y + z = 5$

E. $x - y + z = 5$

2. A particle starts at the origin with initial velocity $\vec{i} + \vec{j} - \vec{k}$. Its acceleration is $\vec{a}(t) = 6t\vec{i} + 2\vec{j} + 6t\vec{k}$. Find its position at $t = 1$.

A. $\frac{1}{6}\,\vec{i} + \frac{1}{2}\,\vec{j} + \frac{1}{3}\,\vec{k}$

B. $\frac{7}{6}\,\vec{i} + \frac{1}{2}\,\vec{j} - \frac{5}{6}\,\vec{k}$

C. $3\vec{i} + 3\vec{j} - 5\vec{k}$

D. $\vec{i} + 2\vec{j} - \vec{k}$

E. $2\,\vec{i} + 2\vec{j} + 0\vec{k}$

3. Find the arc length of the curve defined by $\vec{r}(t) = (\cos(t), \sin(t), 2t), -\pi \le t \le \pi$.

A. π

B. 2π

C. $2\sqrt{3}\pi$

D. $2\sqrt{5}\pi$

E. $2\sqrt{7}\pi$

4. Find a parametric equation for the tangent line to the curve

$$\vec{r}(t) = (3t + 2, t^2, \ln(t))$$

at $t = 1$**.**

A. $x = 3t \quad y = 2t \quad z = 1 + t$

B. $x = 5 + 3t \quad y = 1 + 2t \quad z = t$

C. $x = 3 + 2t, \;\; y = e^t(\cos t - \sin t), \;\; z = \dfrac{1}{t+1}$

D. $x = 3 + 2t \quad y = 1 + t \quad z = 1$

E. $x = 2 - t \quad y = 1 + t \quad z = 3 - 3t$

5. If $L = \displaystyle\lim_{(x,y,z)\to(0,3,4)} \frac{x + 5y - 5z}{\sqrt{x^2 + y^2 + z^2}}$**, then**

A. $L = -3$

B. $L = -2$

C. $L = -1$

D. $L = 0$

E. the limit does not exist

6. If $f(x, y) = \ln(x + 2y^2)$**, then the partial derivative** f_{xy} **equals**

A. $\dfrac{-2x}{(x + 2y^2)^2}$

B. $\dfrac{-4y}{(x + 2y^2)^2}$

C. $\dfrac{4xy}{(x + 2y^2)^2}$

D. $\dfrac{-8xy}{(x + 2y^2)^2}$

E. $\dfrac{4(x^2 - y^2)}{(x + 2y^2)^2}$

7. **Find the unit tangent vector T of $\vec{r}(t) = (\sin(3t))\vec{i} + (\cos(3t))\vec{j} + (4t)\vec{k}$ at any t.**

A. $\mathbf{T} = \frac{3}{5}\cos(3t)\vec{i} - \frac{3}{5}\sin(3t)\vec{j} + \frac{4}{5}\vec{k}$

B. $\mathbf{T} = \frac{3}{5}\sin(3t)\vec{i} - \frac{3}{5}\cos(3t)\vec{j} + \frac{4}{5}\vec{k}$

C. $\mathbf{T} = 3\cos(3t)\vec{i} - 3\sin(3t)\vec{j} + 4\vec{k}$

D. $\mathbf{T} = \sin(3t)\vec{i} - \cos(3t)\vec{j} + 4t\vec{k}$

E. $\mathbf{T} = 1$

8. **Find the curvature of the curve defined by $\vec{r}(t) = (\sin(3t))\vec{i} + (\cos(3t))\vec{j} + (4t)\vec{k}$ at $t = 2$. Recall: $\kappa = |\frac{dT}{ds}| = |\frac{dT}{dt}|/|\mathbf{v}|$**

A. $\frac{3}{5}$

B. $\frac{3}{4}$

C. $\frac{3}{25}$

D. $\frac{9}{25}$

E. 9

9. **Find $\dfrac{\partial z}{\partial y}$ at $(-2, 2, 2)$ if $z(x, y)$ is defined by the equation**

$$xe^y + ye^z = 0$$

A. -1

B. $-\frac{1}{2}$

C. 0

D. $\frac{1}{2}$

E. 1

10. Find a vector $\vec{a}$ and a vector $\vec{b}$ such that the following does not hold

$$|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \cdot |\cos\theta|.$$

(You need to specify $\vec{a}$ and $\vec{b}$, and calculate $|\vec{a} \times \vec{b}|$ and $|\vec{a}| \cdot |\vec{b}| \cdot |\cos\theta|$.)

11. Let C be the intersection of $x^2 + y^2 = 16$ and $x + y + z = 5$. Find a parametric equation for C.