MA 174: Multivariable Calculus

Final EXAM

NAME ______________________ Class Meet Time ______________________

NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

The formula for surface integral, Green's theorem, Stokes' theorem and divergence theorem are attached at the end.

Points awarded

1. (5 pts) ____________	**11. (5 pts)** ____________
2. (5 pts) ____________	**12. (5 pts)** ____________
3. (5 pts) ____________	**13. (5 pts)** ____________
4. (5 pts) ____________	**14. (5 pts)** ____________
5. (5 pts) ____________	**15. (5 pts)** ____________
6. (5 pts) ____________	**16. (5 pts)** ____________
7. (5 pts) ____________	**17. (5 pts)** ____________
8. (5 pts) ____________	**18. (5 pts)** ____________
9. (5 pts) ____________	**19. (5 pts)** ____________
10. (5 pts) ____________	**20. (5 pts)** ____________

Total Points: ____________/100

1. What is the corresponding cylindrical coordinate $(r, \theta, z) = (\_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_)$ **for the point** $(x, y, z) = (-2, 2, 2)$**?**

2. The intersection of the surface $y + 4 = (x-2)^2 + (z+2)^2$ **and the** xz**-plane is**

A. a straight line.

B. two straight lines.

C. a circle.

D. a parabola.

E. a hyperbola.

3. Let $\mathbf{F} = f(x, y, z)\,\mathbf{i} + g(x, y, z)\,\mathbf{j} + h(x, y, z)\,\mathbf{k}$ **and** $\nabla = \partial_x\,\mathbf{i} + \partial_y\,\mathbf{j} + \partial_z\,\mathbf{k}$**, which of the followings are NOT defined?**

I. $\nabla\mathbf{F}$

II. $\nabla \cdot \mathbf{F}$

III. $\nabla \times \mathbf{F}$

IV. $\nabla(\nabla \cdot \mathbf{F})$

V. $\nabla \times (\nabla \times \mathbf{F})$

A. I only

B. II and III

C. IV and V

D. I, II and IV

E. III and V

4. Suppose $z = f(x, y)$, where $x = e^t$ and $y = t^2 + 3t + 2$. Given that $\dfrac{\partial z}{\partial x} = x - y$ and $\dfrac{\partial z}{\partial y} = -x$, find $\dfrac{dz}{dt}$ when $t = 0$.

A. -6

B. -4

C. 6

D. 9

E. 15

5. The function $f(x, y) = y \sin(x)$ has

A. infinitely many local maximum points.

B. infinitely many local minimum points.

C. infinitely many saddle points.

D. exactly one local minimum point and one maximum point.

E. no critical point.

6. Find the minimum value of $x^2+y^2+z^2$ subject to the constraint $2x+y-z-6 = 0$.

A. $\dfrac{25}{6}$

B. 2

C. 4

D. 6

E. 16

7. Find a parametric equation for the line passing through $P = (5, 2, 0)$, and normal to the tangent plane of

$$y^2 + z^2 = 4$$

at P.

A. $x = 0, y = t, z = 0$

B. $x = 5, y = 4t, z = 3t$

C. $x = 5t, y = 2t, z = 3t$

D. $x = 5, y = 4t + 2, z = 0$

E. $x = 5t + 5, y = 2t + 2, z = 3t$

8. The arclength of the curve $\mathbf{r}(t) = \cos(3t)\,\mathbf{i} + \sin(3t)\,\mathbf{j} + 4t\,\mathbf{k}$ for $1 \leq t \leq 3$ is:

A. $\sqrt{2}/4$

B. $\sqrt{3}/4$

C. 2

D. 5

E. 10

9. Evaluate $\displaystyle\int_0^\pi \int_x^\pi \frac{\sin(y)}{y} dy\, dx$.

A. -1

B. 0

C. 1

D. 2

E. $\dfrac{\pi^2}{2}$

10. A sheet of metal occupies the region bounded by the x–axis and the parabola $y = 4 - x^2$. At each point, the density is equal to the distance from the y–axis. Find the mass of the sheet.

A. 16

B. 8

C. 6

D. 4

E. 1

11. Let $\mathbf{F} = x^2y\,\mathbf{i} + xy^2\,\mathbf{j}$. If C is the straight line joining the points $(1, 1)$ and $(2, 2)$, then $\int_C 2\vec{F} \cdot d\mathbf{r} =$

A. 15

B. 10

C. 5

D. -5

E. -10

12. Let C be the boundary of the triangle with vertices $(0, 0)$, $(1, -1)$ and $(2, 1)$ oriented counterclockwise. Then $\int_C -ydx + xdy =$

A. 5

B. 4

C. 3

D. 2

E. 1

13. What is the curvature of a straight line?

A. depend on the location of the line

B. not defined

C. ∞

D. 0

E. 1

14. The directional derivative of the function $f(x,y,z) = \frac{1}{2}x^2y^2z^4$ in the direction of greatest increase at the point $(2,-1,1)$ is

A. $3\sqrt{10}$

B. $2\sqrt{21}$

C. $\sqrt{79}$

D. 9

E. 6

15. If $\mathbf{F}(x,y,z) = (x^2+y^2)\,\mathbf{i} + (y^2+z^2)\,\mathbf{j} + xyz\,\mathbf{k}$ and $\nabla = \partial_x\,\mathbf{i} + \partial_y\,\mathbf{j} + \partial_z\,\mathbf{k}$, then $\nabla(\nabla\cdot\mathbf{F})$ evaluated at $(0,2,\pi)$ equals to

A. $\pi\,\mathbf{i} - \mathbf{j} + \mathbf{k}$

B. $2\,\mathbf{i} - \mathbf{j} - \mathbf{k}$

C. $2\,\mathbf{i} - \pi\,\mathbf{j} + \mathbf{k}$

D. $2\,\mathbf{i} - \mathbf{j} + \pi\,\mathbf{k}$

E. $4\,\mathbf{i} + 2\,\mathbf{j}$

16. $\mathbf{F} = 2xy\,\mathbf{i} + (x^2+3y^2)\,\mathbf{j}$ **is a conservative vector field, i.e.,** $\mathbf{F} = \nabla f$**. Find a potential function** $f(x,y)$**.**

A. $f(x,y) = 2xy$

B. $f(x,y) = x^2y + 3y^2 + 99$

C. $f(x,y) = 2xy + x^2 + 3y^2$

D. $f(x,y) = 2x^2y + (x^2+3y^2)y$

E. $f(x,y) = x^2y + y^3 + 174$

17. Find the flux of $\mathbf{F}(x,y,z) = 5x^3\,\mathbf{i} + 5y^3\,\mathbf{j} + 5z^3\,\mathbf{k}$ **outward through the surface of the cube cut from the first octant by the planes** $x = 2, y = 2,$ **and** $z = 2$**.**

A. 720

B. 480

C. 150

D. -210

E. -30

18. Let D be the solid region bounded by the surfaces S: $x^2+z^2=4,\ y=1,\ y=0$. If $\mathbf{F}(x,y,z)=x^3\,\mathbf{i}+y^3\,\mathbf{j}+z^3\,\mathbf{k}$, then with n being the unit outward normal, evaluate $\displaystyle\iint_S \mathbf{F}\cdot\mathbf{n}\ d\sigma$.

A. 8π

B. $\dfrac{28}{3}\ \pi$

C. 28π

D. 10π

E. 20

19. Find the surface area of the part of the surface $z=\dfrac{1}{6}(x^2+y^2)$ below the plane $z=\dfrac{8}{3}$.

A. $\dfrac{106}{3}\pi$

B. $\dfrac{215}{11}\pi$

C. $\dfrac{\pi}{6}(37^{3/2}-1)$

D. $\dfrac{\pi}{6}(29^{3/2}-1)$

E. $\dfrac{196}{9}\pi$

20. Evaluate the circulation of the field $\mathbf{F} = 6y\,\mathbf{i} + 6xz\,\mathbf{j} + 6x^2\,\mathbf{k}$ around boundary of the triangle cut from the plane $x+y+z = 1$ by the first octant, counterclockwise when viewed from above.

A. -5

B. -3

C. 8

D. 6

E. $-\dfrac{5}{3}$

Surface Integral:

If R is the shadow region of a surface S defined by the equation $f(x, y, z) = c$, and g is a continuous function defined at the points of S, then the integral of g over S is the integral

$$\int\int_S g(x,y,z)\,d\sigma = \int\int_R g(x,y,z)\frac{|\nabla f|}{|\nabla f \cdot \mathbf{p}|}dA,$$

where p is a unit vector normal to R and $|\nabla f \cdot \mathbf{p}| \neq 0$.

Green's Theorem:

$$\oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

where C is a positively oriented simple closed curve enclosing region R, and P, Q have continuous partial derivatives.

Divergence Theorem:

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S \mathbf{F} \cdot \mathbf{n}\,d\sigma = \iiint_D \nabla \cdot \mathbf{F}\,dV$$

where D is a simple solid region with boundary S given outward orientation, and component functions of F have continuous partial derivatives.

Stokes' Theorem:

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \nabla \times \mathbf{F} \cdot \mathbf{n}\ d\sigma$$

where C, given counterclockwise direction, is the boundary of oriented surface S, n is the surface's unit normal vector and component functions of F have continuous partial derivatives.