MA 174

Final Exam

Prof. Naughton
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Only scientific calculators without graphing capabilities are allowed.

Name in capitals:

1. Let $f(x, y) = x^2 + y^2 + x^2y + 4$. Find all critical points of f and classify them as local maximums, local minimums, or saddle points.

2. True or false. Justify your answer.

   a) Any constant vector field is conservative.

   b) The binormal vector is $\vec{B}(t) = \vec{N}(t) \times \vec{T}(t)$.

   c) There exists a function f with continuous second order partial derivatives such that $f_x(x, y) = x + y^2$ and $f_y(x, y) = x - y^2$.

3. Find an equation of the tangent plane to the surface $x\cos(z) - y^2\sin(xz) = 2$ at P(2,1,0).

4. A particle has acceleration $\vec{a}(t) = \langle 2t, 2, 0 \rangle$ m/s/s.

a) Find its velocity if the initial velocity is $\vec{v}(0) = \langle 3, 0, 4 \rangle$ m/s.

b) What is the distance travelled in the first five seconds?

5. Let D be the region inside the sphere with center (0,0,0) and radius 2, outside the cylinder $x^2 + y^2 = 1$, satisfying $x - y \geq 0$, and $y \geq 0$ Express $\int\int\int_D yz dV$ as one or more iterated integrals using first a) cylindrical coordinates, and then b) spherical coordinates. Do not evaluate the integrals.

6. a) Find the rate of change of $f(x, y) = 3x^2y + y^3$ at (1,2) in the direction $\langle -3, 4 \rangle$.

   b) In what direction should one move from (1,2) so that f decreases fastest?

   c) Use the total differential of f to estimate the change in f in moving from (1,2) to (0.9,2.2).

7. Find the absolute maximum and the absolute minimum of the function $f(x, y) = 5 - 3x + 4y$ on the closed triangular region with vertices (0,0), (4,0), and (4,5).

8. Let D be the region bounded by the planes $x + 2y + z = 2$, $2y = x$, $x = 0$, and $z = 0$ with density function $\delta(x, y, z) = 1$. Find the moment of inertia of the region D about the y-axis Sketch the region and its projection. Do not evaluate the integral.

9. Find the circulation of $\vec{F}(x, y, z) = \langle y + x^2, x, 3xz + y \rangle$ around C, the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the plane $z = x + 2y$ with counterclockwise orientation when viewed from above.

10. Let $\vec{F}(x, y, z) = \langle 3x^2yz - 3y, x^3z - 3x, x^3y + 2z \rangle$ Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is the curve with initial point (0,0,2) and terminal point (0,3,0) shown in the figure.

11. Use Green's Theorem to evaluate $\oint_C x^2 y dx + (y + xy^2) dy$,where C is the boundary of the region enclosed by $y = x^2$ and $x = y^2$.

12. Find the outward flux through the sphere with center (0,0,0) and radius 5 of the vector field $\vec{F}(x, y, z) = \langle x + e^{y^2+z^2}, y + e^{z^2+x^2}, z + e^{x^2+y^2} \rangle$.