MA 261

Alternate Test 1

Note: Be neat, be organized, and show all your work.

Formulas:

$$a_{\mathbf{T}} = \frac{\mathbf{R}' \cdot \mathbf{R}''}{|\mathbf{R}'|}, \quad a_{\mathbf{N}} = \frac{|\mathbf{R}' \times \mathbf{R}''|}{|\mathbf{R}'|}, \quad \kappa = \frac{|\mathbf{R}' \times \mathbf{R}''|}{|\mathbf{R}'|^3}, \quad |\mathbf{R}''|^2 = a_{\mathbf{T}}^2 + a_{\mathbf{N}}^2.$$

(1) (15 points) Parameterize by arclength the curve

$$\mathbf{R}(t) = t\,\mathbf{i} + \cos t\,\mathbf{j} + \sin t\,\mathbf{k}, \qquad 0 \le t \le \pi.$$

(2) An object moves along a path given by

$$\mathbf{R}(t) = 2\sin t\,\mathbf{i} + 2\cos t\,\mathbf{j} + \sin 2t\,\mathbf{k}, \qquad 0 \le t \le 20.$$

(a) (15 points) For which values of t does this object experience the greatest acceleration? the least acceleration? (b) (5 points) Is this curve closed? (Yes or no.) (c) (5 points) Is it simple?

(3) (5 points) Find $\mathbf{T}(t)$ for the curve

$$\mathbf{R}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + 2t\,\mathbf{k}, \qquad -\infty < t < \infty$$

(10 points) Find the line tangent to the curve at the point $\mathbf{R}(1)$, i.e., when $t = 1$. (10 points) Calculate the curvature of the curve at the point $\mathbf{R}(1)$. (5 points) This curve lies in a plane. Find an equation for that plane. (Note: if you're finding the plane with a long calculation, then work another problem for 5 points—the equation for the plane can be found with almost no calculation.)

(4) (10 points) In the region $-2 \le x \le 2$, graph the curves

$$\mathbf{R}(t) = t\,\mathbf{i} + (t^2 - 1)\,\mathbf{j}, \quad -\infty < t < \infty, \text{ and}$$
$$\mathbf{R}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j}, \quad 0 \le t \le 2\pi.$$

(10 points) Find the cosines of the angles between these curves at the points $(x, y) = (0, -1)$ and $(x, y) = (1, 0)$.

(5) (10 points) Find the plane perpendicular to the curve

$$\mathbf{R}(t) = e^t\,\mathbf{i} + t^2\,\mathbf{j} + \frac{1}{1+t^2}\,\mathbf{k}, \qquad -\infty < t < \infty,$$

at the point $\mathbf{R}(1)$, i.e., when $t = 1$.