TEST 1
MA 261

MWF 2:30 B. Lucier

(1) (20 points) Assume that M, m, and G are positive numbers. (One can think of M as the mass of the sun, m as the mass of a satellite (planet, asteroid, etc.) about the sun, and G as the gravitational constant.) Show that the vector function

$$\mathbf{r}(t) = (a\cos bt)\mathbf{i} + (a\sin bt)\mathbf{j}$$

satisfies Newton's law of gravitation,

$$m\mathbf{r}''(t) = -\frac{GmM}{|\mathbf{r}(t)|^2}\frac{\mathbf{r}(t)}{|\mathbf{r}(t)|},$$

if and only if Kepler's law holds:

$$a^3b^2 = GM.$$

(2) (30 points) Find the unit tangent $\mathbf{T}(t)$, the principal unit normal $\mathbf{N}(t)$, and the curvature $\kappa(t)$ of the curve

$$\mathbf{r}(t) = (\ln\sec t)\mathbf{i} + t\mathbf{j}, \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

(Note: $\sec t = 1/\cos t$.)

(3) (15 points) Assume that two parents are putting together a saving plan for their children to go to college, and that they will put d dollars per year in a savings account that will earn interest at a rate of $100I$ percent a year. Then after t years they can expect a total amount of

$$T(I,d,t) = \frac{d}{I}(e^{It} - 1)$$

dollars in the account. Derive an expression that will estimate the change in the total after 10 years if I is decreased by 0.02 (i.e., 2%) and d is decreased by \$100.

(4) (15 points) Graph the surface given by the equation

$$x^2 - y^2 + z^2 = 1.$$

(5) (20 points) Parametrize the curve given by the intersection of the two surfaces

$$x^2 + y^2 - 2z - 5 = 0 \quad \text{and} \quad x^2 + y^2 + z^2 = 4.$$