TEST 2
MA 261

October 29, 1991 MWF 2:30 PM

(1) Any electric potential (voltage) $V(x, y)$ in a vacuum satisfies

(1)
$$\frac{\partial^2 V}{\partial x^2}(x, y) + \frac{\partial^2 V}{\partial y^2}(x, y) = 0,$$

and if a function satisfies (1) then it is an electric potential.

(a) (10 points) Show that the function $V(x, y) = x^2 - y^2$ satisfies (1), and hence is a valid electric potential

(b) (10 points) The electric field $\mathbf{E}(x, y)$ (which is a vector field) associated with a potential $V(x, y)$ is given by

$$\mathbf{E}(x, y) = \nabla V(x, y).$$

Graph $\mathbf{E}(x, y)$ for $-1 \le x \le 1$, $-4 \le y \le 4$, when $V(x, y) = x^2 - y^2$.

(c) (10 points) Assume that $V(x, y) = x^2 - y^2$ is the electric potential in a strip $-1 \le x \le 1$ and $-\infty < y < \infty$. Assume that a charged particle with mass m and charge e is put into the strip. Then the location $\mathbf{r}(t) = (r_1(t), r_2(t))$ of the charged particle satisfies the equation

$$m\mathbf{r}''(t) = e\mathbf{E}(r_1(t), r_2(t)).$$

Assuming that $m > 0$ and $e > 0$, where will a particle initially placed at the point $(1/2, 0)$ hit the boundary $x = 1$? (Hint: Look where the electric field $\mathbf{E}$ will push the particle.)

(d) (10 points) Under the assumptions of Part (c), is there any point $\mathbf{r}(0) = (r_1(0), r_2(0))$ in the strip where one can place a charged particle and it will not move, i.e., where $\mathbf{E}(r_1(0), r_2(0)) = \mathbf{0}$?

(e) (10 points) Any point (x, y) where $\mathbf{E}(x, y) = \mathbf{0}$ is a critical point of the function V. Find all critical points of $V(x, y) = x^2 - y^2$ in the strip $-1 < x < 1$ and $-\infty < y < \infty$. Classify these critical points as maxima, minima, or saddle points.

(2) (25 points) Find all maxima and minima of the function $f(x, y) = e^{xy}$ subject to the constraint $x^2 + y^2 \le 1$.

(3) (15 points) Evaluate

$$\lim_{(x,y)\to 0} \frac{x^2 + x^2y^2 + y^2}{x^2 + y^2}.$$

(4) (20 points) Graph the level curves of the function $f(x, y) = (x^2 + y^2)/x$. Why do the level curves show you that $\lim_{(x,y)\to(0,0)} f(x, y)$ does not exist?