MA 271 Vector Calculus
Fall 1999, Test One

Instructor: Yip

- This test booklet has SIX QUESTIONS, totaling 60 points for the whole test. You have 50 minutes to do this test. **Plan your time well. Read the questions carefully.**
- This test is closed book and closed notes.
- (Any kind of) calculator is allowed. But you should **not** use it whenever it is possible (from the point of view of this class), i.e. your answers should be as **analytical** as possible.
- In order to get full credits, you need to give **correct** and **simplified** answers and explain in a **comprehensible way** how you arrive at them.
- You can use both sides of the papers to write your answers. But please indicate so if you do.

Name:_____

Question	Score
1.(10 pts)	
2.(10 pts)	
3.(10 pts)	
4.(10 pts)	
5.(10 pts)	
6.(10 pts)	
Total (60 pts)	

1. Find the area of the triangle with vertices:

$$A = (2, -1, 5), \quad B = (3, 2, -1), \quad C = (-1, 2, 2)$$

2. Given the point $P_0 = (5, 0, 12)$ and the plane $\Pi : x - 2y + 5z = 5$. Find the point P_1 on Π such that $|P_0P_1|$ is the shortest distance between P_0 and Π.

3. Given the following two lines:

$$L_1: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \\ 4 \end{pmatrix} + t \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$$

$$L_2: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \\ -3 \end{pmatrix} + s \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}$$

Find the point Q_1 on L_1 and Q_2 on L_2 such that $|Q_1Q_2|$ gives the shortest distance between the two given lines.

4. Determine the convergence/divergence of the following series. *Clearly indicate your reasons.*

$$(a) \quad \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

$$(b) \quad \sum_{n=1}^{\infty} \frac{n+2^n}{n^2 2^n}$$

$$(c) \quad \sum_{n=1}^{\infty} (-1)^n (\sqrt{n^2+n} - n)$$

5. Given the following power series:

$$P(x) = 1 - \frac{x}{(1!)^2} + \frac{x^2}{(2!)^2} - \frac{x^3}{(3!)^2} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{(n!)^2}$$

(a) Determine the region of convergence for $P(x)$.

(b) What is the value of $P(1)$ up to 4 decimal places?

6. Given the following series:

$$Q(x) = \sum_{n=0}^{\infty} \left(\frac{x^2 + 1}{3} \right)^n$$

(a) Determine the region of convergence for $Q(x)$.

(b) Find $Q(x)$, i.e. express $Q(x)$ as an explicit function.