# MA 271: Several Variable Calculus

## EXAM II

**Oct. 30, 2007**

NAME ____________________ Lecture Time ____________________

**NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.**

**Points awarded**

| | | | |
|---|---|---|---|
| **1. (6 pts)** | ________ | **10. (6 pts)** | ________ |
| **2. (6 pts)** | ________ | **11. (6 pts)** | ________ |
| **3. (6 pts)** | ________ | **12. (6 pts)** | ________ |
| **4. (6 pts)** | ________ | **13. (6 pts)** | ________ |
| **5. (6 pts)** | ________ | **14. (6 pts)** | ________ |
| **6. (6 pts)** | ________ | **15. (6 pts)** | ________ |
| **7. (6 pts)** | ________ | **16. (6 pts)** | ________ |
| **8. (6 pts)** | ________ | **17. (6 pts)** | ________ |
| **9. (6 pts)** | ________ | | |

**Total Points:** ________/102

**1. Use the degree two Taylor polynomial of $\ln(x)$ centered at $x_0 = 1$ to estimate**

$$I = \int_{0.7}^{1.3} \ln(x)dx.$$

**The approximate value of $I$ is**

**A.** $0$

**B.** $0.6(\ln(1.3) - \ln(0.7))$

**C.** $0.008$

**D.** $\boxed{-0.009}$

**E.** $-0.012$

**2. If $L = \lim_{(x,y)\to(0,0)} \frac{x}{\sqrt{x^2+y^2}}$, then**

**A.** $L = -3$

**B.** $L = -2$

**C.** $L = -1$

**D.** $L = 0$

**E.** $\boxed{\textbf{the limit does not exist}}$

**3. Find $f_x(0,0)$ when**

$$f(x,y) = \begin{cases} \frac{x^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

**Recall:**

$$f_x(x_0,y_0) = \lim_{\Delta x\to 0} \frac{f(x_0+\Delta x, y_0) - f(x_0,y_0)}{\Delta x}$$

**A.** $L = -3$

**B.** $L = -2$

**C.** $L = -1$

**D.** $L = 0$

**E.** $\boxed{\textbf{the limit does not exist}}$

**4. If $f(x,y)=\ln(x^2+2y^2)$, then the partial derivative $f_{xy}$ equals**

A. $\boxed{\dfrac{-8xy}{(x^2+2y^2)^2}}$

B. $\dfrac{4xy}{(x^2+2y^2)^2}$

C. $\dfrac{4(x^2-y^2)}{(x^2+2y^2)^2}$

D. $\dfrac{-4y}{(x^2+2y^2)^2}$

E. $\dfrac{-2x}{(x^2+2y^2)^2}$

**5. If $xz+y\ln x-x^2+4=0$ defines $x$ as a function of two independent variables $y$ and $z$, find $\dfrac{\partial x}{\partial z}$ at $(x,y,z)=(1,-1,-3)$.**

A. $0$

B. $\boxed{\frac{1}{6}}$

C. $\frac{1}{5}$

D. $\frac{1}{3}$

E. $\frac{1}{2}$

**6. The directional derivative of $f(x,y)=x^3e^{-2y}$ in the direction of greatest increase of $f$ at the point $(1,0)$ is**

A. $\sqrt{5}$

B. $\boxed{\sqrt{13}}$

C. 5

D. 6

E. 13

**7. In what direction is the derivative of $z = x^2 + 3xy - \frac{1}{2}y^2$ at $(-1,-1)$ equal to zero.**

**A.** $3\vec{\imath}$

**B.** $5\vec{\imath} + 2\vec{\jmath} - \vec{k}$

**C.** $\boxed{2\vec{\imath} - 5\vec{\jmath}}$

**D.** $2\vec{\imath} + 5\vec{\jmath}$

**E.** $\sqrt{29}$

**8. Find an equation for the tangent plane of $z = x^2 + 3xy - \frac{1}{2}y^2$ at $(-1,-1)$.**

**A.** $5x + 2y + z - \frac{7}{2} = 0$

**B.** $5x + 2y - z - 2 = 0$

**C.** $\boxed{5x + 2y + z + \frac{7}{2} = 0}$

**D.** $5x - 2y + z - \frac{7}{2} = 0$

**E.** $5x - 2y - z + \frac{7}{2} = 0$

**9. By using a linear approximation of $f(x,y) = x^2 - x + \sin(y)$ at $(2,0)$, compute the approximate value of $f(1.9, 0.1)$.**

**A.** $\boxed{1.8}$

**B.** 1.85

**C.** 1.9

**D.** 2

**E.** 3.5

**10. Find $\dfrac{\partial z}{\partial u}$ if $z = x^2 + xy^3$ where $x = uv^2 + w^3$ and $y = u + ve^w$ at $u = 2, v = 1, w = 0$.**

**A.** 30

**B.** 42

**C.** 53

**D.** 64

**E.** $\boxed{85}$

**11. Find the largest product the positive numbers $x$, $y$ and $z$ can have if**

$$x + y + z^2 = 5.$$

**A.** 0

**B.** 3

**C.** $\boxed{4}$

**D.** 5

**E.** $\infty$

**12. Find $\left(\dfrac{\partial w}{\partial x}\right)_y - \left(\dfrac{\partial w}{\partial x}\right)_z$ at $(w, x, y, z) = (6, 1, 1, 2)$ if**

$$w = x^2 + y^2 + z^2, \quad z = x^2 + y^2$$

**A.** $-4$

**B.** 0

**C.** 4

**D.** $\boxed{10}$

**E.** 12

**13. Evaluate** $\displaystyle\int_3^0 \int_1^{-1} x^2 + xy^2 dy\, dx.$

**A.** $\frac{38}{3}$

**B.** $\frac{35}{2}$

**C.** $24$

**D.** $\boxed{21}$

**E.** $-\frac{7}{2}$

**14. Evaluate** $\displaystyle\int_0^1 \int_y^1 x^2 e^{xy} dx\, dy.$

**A.** $\frac{e+2}{2}$

**B.** $\frac{e}{2}$

**C.** $1$

**D.** $\boxed{\frac{e-2}{2}}$

**E.** $\dfrac{e^{xy}}{2}$

**15. Find** **<u>all</u>** **the values of** $x$**, such that the following series** **<u>converges</u>** **(do not forget the end values)**

$$\sum_{n=1}^{\infty} (-1)^n \frac{(n+1)(x+5)^n}{n^2}.$$

**Your Answer: For** $x$ **satisfying** ____________________**, the series converges.**

**Answer:** $-6 < x \leq -4$

**16. Find the degree three Taylor polynomial of $f(x,y) = e^x \ln(y)$ near $(x_0, y_0) = (0,1)$.**

**Answer:** $P_3(x,y) = (y-1) - \frac{(y-1)^2}{2} + x(y-1) + \frac{(y-1)^3}{3} - \frac{x(y-1)^2}{2} + x^2(y-1)$

**17. Find and classify (local maximum, local minimum, saddle) <u>all</u> the critical points of**

$$f(x,y) = 2x^3 + xy^2 + 6x^2 + y^2$$

**Answer:** $(0,0)$ **local minimum,** $(-2,0)$ **local max,** $(-1, \pm\sqrt{6})$ **saddle**