MA 271 Vector Calculus
# Fall 1999, Test Three

Instructor: Yip

- This test booklet has SIX QUESTIONS, totaling 60 points for the whole test. You have 50 minutes to do this test. **Plan your time well. Read the questions carefully.**

- This test is closed book and closed notes.

- (Any kind of) calculator is allowed. But you should **not** use it whenever it is possible (from the point of view of this class), i.e. your answers should be as **analytical** as possible.

- In order to get full credits, you need to give **correct** and **simplified** answers and explain in a **comprehensible way** how you arrive at them.

- You can use both sides of the papers to write your answers. But please indicate so if you do.

Name:_____

Question	Score
1.(10 pts)	
2.(10 pts)	
3.(10 pts)	
4.(10 pts)	
5.(10 pts)	
6.(10 pts)	
Total (60 pts)	

1. Alice is given the following map to wonder around..... For each of the points A, B, C, D, E, F on the map,

   (a) classify whether it is a local-minimum/local-maximum/saddle point or none of the above.

   (b) *roughly draw* the gradient vector ∇h of the height function at these points.

Map of Alice in Her Wonderland
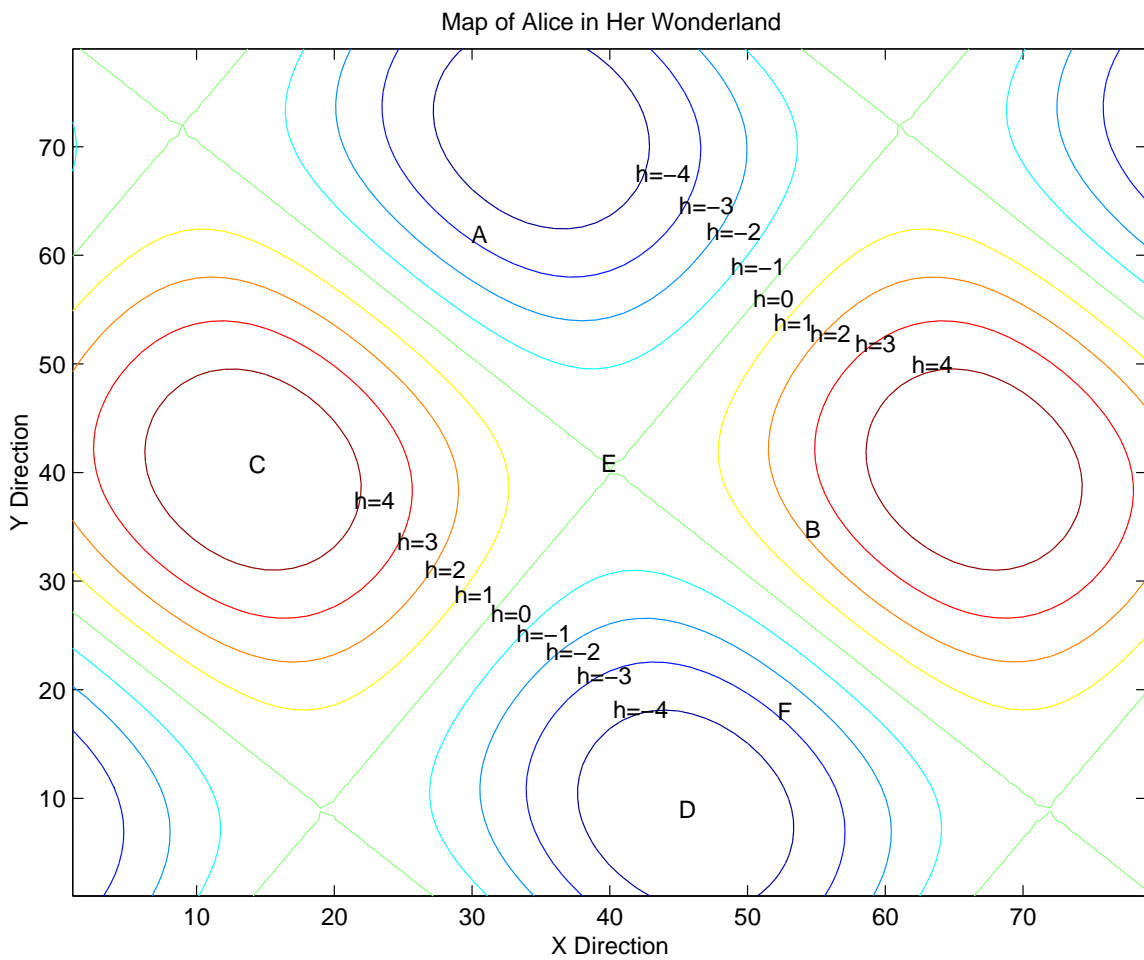
2. Find the equation of the straight lines *tangent* and *normal* to the *level curve* of the following function at the point $(3, -2)$:

$$f(x, t) = x^3 + xy.$$

3. An ant is crawling on the curve: $x^2 + 4y^2 = 4$. The background temperature field on the region is given by: $T(x, y) = xy$.

What are the maximum and minimum temperatures felt by the ant?

4. Find the center of mass of the following domain (of unit density) in R^2:

$$\frac{x^2}{9} + \frac{y^2}{16} \le 1, \quad x \ge 0, \ y \ge 0$$

(Hint: The area of the region bounded by the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is $\pi a b$.)

5. Find the volume of the following solid in R^3:

$$x^2 + y^2 \leq z \leq 8 - x^2 - y^2.$$

(Hint: Draw a picture.)

6. Find the center of mass of the following $CURVE$ (of unit density):

$$x^2 + y^2 = 1, \quad x \geq 0, \ y \geq 0$$