

1. For what values of α is the angle between $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{i} + \alpha\mathbf{k}$ equal to $\pi/4$?

- A) 0 B) $-1 \pm \sqrt{6}$ C) $-2 \pm \sqrt{6}$ D) $-1 \pm \sqrt{3}$ E) $-2 \pm \sqrt{3}$

2. Let \vec{u} and \vec{v} be vectors and c be a scalar. Which of the following are not always true?

- A) $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
- B) $\vec{u} \cdot \vec{u} = |\vec{u}|^2$
- C) $(\vec{u} \times \vec{v}) \cdot \vec{u} = \vec{v} \cdot (\vec{u} \times \vec{v})$
- D) $c(\vec{u} \times \vec{v}) = c\vec{v} \times c\vec{u}$
- E) $(\vec{u} \times \vec{u}) \cdot \vec{u} = 0$

3. Find the speed of the particle with position function $\vec{r}(t) = e^{3t} \mathbf{i} + e^{-3t} \mathbf{j} + te^{3t} \mathbf{k}$ when $t = 0$.

A) $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

B) 1

C) $\sqrt{2}$

D) $\sqrt{17}$

E) $\sqrt{19}$

4. The plane S passes through the point $P(1, 2, 3)$ and contains the line $x = 3t$, $y = 1 + t$, and $z = 2 - t$. Which of the following vectors is normal to S ?

A) $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

B) $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

C) $\mathbf{i} + \mathbf{k}$

D) $\mathbf{i} - 2\mathbf{j}$

E) $\mathbf{i} + 2\mathbf{j}$

5. Let $z = e^r \cos \theta$, $r = 10st$, $\theta = \sqrt{s^2 + t^2}$. The partial derivative $\frac{\partial z}{\partial s}$ is:

A) $e^r \left(10t \cos \theta - \frac{s \sin \theta}{\sqrt{s^2 + t^2}} \right)$

B) $e^r \left(t \cos \theta - \frac{s \sin \theta}{\sqrt{s^2 + t^2}} \right)$

C) $e^r \left(10t \cos \theta + \frac{s \sin \theta}{\sqrt{s^2 + t^2}} \right)$

D) $e^r \left(t \cos \theta + \frac{s \sin \theta}{\sqrt{s^2 + t^2}} \right)$

E) $10t \cos \theta - \frac{se^r \sin \theta}{\sqrt{s^2 + t^2}}$

6. The direction in which $f(x, y) = x^2y + e^{xy} \sin y$ increases most rapidly at $(1, 0)$ is:

A) \mathbf{i}

B) \mathbf{j}

C) $\frac{1}{\sqrt{2}} \mathbf{i} + \frac{1}{\sqrt{2}} \mathbf{j}$

D) $\frac{1}{\sqrt{5}} \mathbf{i} + \frac{2}{\sqrt{5}} \mathbf{j}$

E) $\frac{2}{\sqrt{13}} \mathbf{i} + \frac{3}{\sqrt{13}} \mathbf{j}$

7. The equation of the tangent plane to $z + 5 = xe^y \cos z$ at $(5, 0, 0)$ is:

A) $x + 5y + z = 5$

B) $x + y - 5z = 5$

C) $5x + y - z = 5$

D) $x + 5y - z = 5$

E) $x + y - z = 5$

8. The maximum value of $f(x, y) = 8x^2 - 4y^2$ under the constraint $8x^2 + 4y^2 = 9$ is:

A) $f(x, y) = 1/9$

B) $f(x, y) = 1/4$

C) $f(x, y) = 1/8$

D) $f(x, y) = 8$

E) $f(x, y) = 9$

9. Which of the following series is conditionally convergent?

A) $\sum_{n=1}^{\infty} (-1)^n e^n$

B) $\sum_{n=1}^{\infty} (-1)^n e^{-n}$

C) $\sum_{n=1}^{\infty} (-1)^n n^{1/2}$

D) $\sum_{n=1}^{\infty} (-1)^n n^{-1/2}$

E) $\sum_{n=1}^{\infty} (-1)^n \ln n$

10. Use the first three terms of the MacLaurin series for e^{-x^2} to approximate $\int_0^1 e^{-x^2} dx$

A) $\frac{3}{4}$

B) $\frac{23}{30}$

C) $\frac{10}{11}$

D) $\frac{17}{25}$

E) $\frac{5}{7}$

11. Consider the two series

$$\text{I. } \sum_{n=1}^{\infty} (-1)^{n+1} \sin\left(\frac{1}{n}\right)$$

$$\text{II. } \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n(\ln n)^2}$$

- A) both series converge absolutely
- B) series (I) converges absolutely, while series (II) converges conditionally
- C) series (I) converges conditionally, while series (II) converges absolutely
- D) both series converge conditionally
- E) at least one of the series diverges

12. The interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-1)^n}{n3^n}$$

is of the form

- A) $[1-r, 1+r]$
- B) $[1-r, 1+r)$
- C) $(1-r, 1+r]$
- D) $(1-r, 1+r)$
- E) The series converges only at $x = 1$

13. Find the volume of the solid bounded by the planes $x + 2y + 3z = 6$, $y = x$, $y = 0$ and $z = 0$.

A) $\frac{14}{3}$

B) $\frac{16}{3}$

C) $\frac{14}{9}$

D) $\frac{16}{3}$

E) $\frac{15}{4}$

14. Evaluate the double integral by reversing the order of integration

$$\int_0^3 \int_{y^2}^9 y \cos(x^2) dx dy$$

A) $\frac{\sin 81}{8}$

B) $\frac{\sin 9}{16}$

C) $\frac{\sin 9}{8}$

D) $\frac{\sin 81}{4}$

E) $\frac{\sin 81}{16}$

15. Use spherical coordinates to find the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 4$, above the xy -plane, and below the cone $z = \sqrt{x^2 + y^2}$

A) $\frac{8\sqrt{2}\pi}{3}$ B) $\frac{16\sqrt{2}\pi}{3}$ C) $\frac{16\sqrt{2}\pi}{9}$ D) $\frac{8\sqrt{2}\pi}{9}$ E) $\frac{4\sqrt{2}\pi}{3}$

16. Evaluate

$$\iint_R \sqrt{x^2 + y^2} \, dA$$

where R is the region bounded by the graphs of $y = \sqrt{4 - x^2}$, $y = -x$, and $y = x$

A) $\frac{2\pi}{3}$ B) $\frac{4\pi}{3}$ C) $\frac{3\pi}{4}$ D) $\frac{3\pi}{2}$ E) $\frac{\pi}{2}$

17. Evaluate the line integral $\int_C xy^4 ds$, where C is given by

$$\vec{r}(t) = 4 \cos t \mathbf{i} + 4 \sin t \mathbf{j} \quad \text{for} \quad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

18. Evaluate

$$\int_C -y \, dx + x \, dy$$

where C is given by $\vec{r}(t) = \cos t \, \mathbf{i} + \sin t \, \mathbf{j}$ for $0 \leq t \leq 2\pi$.

19. Show that $\vec{F}(x, y) = (6xy + 4)\mathbf{i} + 3x^2\mathbf{j}$ is a conservative vector field and evaluate

$$\int_C (6xy + 4) dx + 3x^2 dy$$

where C is given by

$$\vec{r}(t) = \frac{t+4}{\sqrt{t^3+1}}\mathbf{i} + \sqrt{t^4+9}\mathbf{j} \text{ for } 0 \leq t \leq 2$$

20. Evaluate

$$\iint_S 4x \, d\sigma$$

where S is the surface $2x^2 + y - z = 0$ for $0 \leq x \leq 1$ and $0 \leq y \leq 1$