MA 271
FINAL EXAM INSTRUCTIONS
Dec. 10, 2007

NAME ______________________ INSTRUCTOR ______________________

1. You must use a #2 pencil on the mark–sense sheet (answer sheet).

2. On the mark-sense sheet, fill in the instructor's name (Min Chen or Adrian Jenkins) and the course number which is MA271.

3. Fill in your NAME and blacken in the appropriate spaces.

4. Fill in the SECTION NUMBER boxes with the division and section number of your class and blacken in the appropriate spaces.

- 0101 if your class meets during: 9:30am–10:20am
- 0201 if your class meets during: 10:30am–11:20am
- 0301 if your class meets during: 11:30am–12:20pm
- 0401 if your class meets during: 12:30pm–1:20pm

5. Leave the TEST/QUIZ NUMBER blank.

6. Fill in the 10-DIGIT PURDUE ID and blacken in the appropriate spaces.

7. Sign the mark–sense sheet.

8. Fill in your name and your instructor's name on the question sheets (above).

9. There are 20 questions, each worth 5 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–20 in the answer sheet. Do all your work on the question sheets. Turn in both the mark–sense sheets and the question sheets when you are finished.

10. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

11. NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

1. The series

$$\sum_{n=1}^{\infty}(-1)^n \frac{(n+1)(2x+7)^n}{n^2}$$

converges on an interval of the form

A. $a < x < b$

B. $\boxed{a < x \leq b}$

C. $a \leq x < b$

D. $a \leq x \leq b$

E. $-\infty < x < \infty$

2. The limit as $n \to \infty$ of the sequence

$$a_n = \left(1 + \frac{1}{\sqrt{n}}\right)^n$$

is

A. 0

B. 1

C. e

D. $\sqrt{e}$

E. $\boxed{\infty}$

3. What is the magnitude of the projection of the vector $(1, -2, 7)$ onto the vector $(3, 4, 0)$?

A. $\frac{2}{3}$

B. $\frac{11}{13}$

C. $\boxed{1}$

D. $\frac{7}{5}$

E. 4

4. The position of an object is given by

$$\mathbf{r}(t) = \left(3t, \frac{3}{\sqrt{2}}t^2, t^3\right)$$

with length scale in meters. At what time t has the object traveled 14 meters? (The object started traveling at $t = 0$)

A. 1

B. $\boxed{2}$

C. 3

D. 6

E. 11

5. The position of an object is given by

$$\mathbf{r}(t) = (e^t \cos(t), e^t \sin(t), 2).$$

Find the unit tangent vector $\mathbf{T}$ at $t = \pi$.

A. $\mathbf{T} = \frac{1}{\sqrt{2}}(1, 1, 2)$

B. $\mathbf{T} = \frac{1}{\sqrt{2}}(1, -1, 0)$

C. $\boxed{\mathbf{T} = \frac{1}{\sqrt{2}}(-1, -1, 0)}$

D. $\mathbf{T} = \frac{1}{\sqrt{2}}(-1, 1, 0)$

E. $\mathbf{T} = \frac{1}{\sqrt{2}}(-1, 1, 2)$

6. Let

$$f(x, y) = \begin{cases} 2x + 3y + 1, & xy \neq 0, \\ 0, & xy = 0. \end{cases}$$

Find $f_x(0, 0)$ and $f_x(0, 1)$.

A. $f_x(0, 0) = 0$ and $f_x(0, 1) = 2$

B. $\boxed{f_x(0, 0) = 0 \text{ and } f_x(0, 1) \text{ is not defined}}$

C. $f_x(0, 0) = 2$ and $f_x(0, 1) = 2$

D. $f_x(0, 0) = 2$ and $f_x(0, 1)$ is not defined

E. $f_x(0, 0)$ is not defined and $f_x(0, 1) = 2$

7. The linearization of the function

$$f(x, y, z) = x \sin(yz) + e^z \sin(y^2)$$

at the point $P(1, \sqrt{\pi}, 0)$ is given by

A. $2\pi - \sqrt{\pi}(x + 2y + 3z)$

B. $\pi - \sqrt{\pi}(x - 2z)$

C. $\sqrt{\pi} + 2y - 3z$

D. $2\pi + \sqrt{\pi}(2x + y - z)$

E. $\boxed{2\pi - \sqrt{\pi}(2y - z)}$

8. For what values of the constant k will the Second Derivative Test guarantee that $f(x, y) = x^2 + kxy + y^2$ has a minimum at $(0, 0)$?

A. There is no such k

B. $|k| < \infty$

C. $k < 2$

D. $\boxed{|k| < 2}$

E. $|k| > 2$

9. Find the maximum distance from the sphere

$$x^2 + y^2 + z^2 = 12$$

to the point $(1, -1, 1)$.

A. 1

B. 3

C. $\sqrt{8}$

D. $\boxed{\sqrt{27}}$

E. 6

10. Evaluate

$$\int_0^1 \int_{\sqrt{x}}^1 9\sqrt{3 + y^3}\, dydx.$$

A. $\boxed{2(8 - 3\sqrt{3})}$

B. $2(8 - 3\sqrt{5})$

C. $2(5 - 3\sqrt{3})$

D. $3\sqrt{3}$

E. $5\sqrt{5}$

11. Find the volume of of the region enclosed by the ellipsoid

$$x^2 + (y/2)^2 + (z/3)^2 = 1.$$

A. 2π

B. 4π

C. $\boxed{8\pi}$

D. 10π

E. 16π

12. Find the Jacobian $J(r, t, u)$ of the transformation

$$x = 2r\cos(t), \quad y = 3r\sin(t), \quad z = 4u.$$

A. $\boxed{24\,r}$

B. r

C. r^2

D. $12\,r^2\,u$

E. $6\,r\,\sin(t)$

13. Let $\mathbf{F}(x,y,z) = x^2yz\mathbf{i}+(x^2+y^2+z^2)\mathbf{j}+xy^2z\mathbf{k}$, evaluate $\nabla\cdot(\nabla\times\mathbf{F})$ at $(x,y,z) = (2,-1,3)$.

A. -8

B. -4

C. $\boxed{0}$

D. 1

E. 16

14. Let $\mathbf{F} = f(x,y,z)\,\mathbf{i} + g(x,y,z)\,\mathbf{j} + h(x,y,z)\,\mathbf{k}$ and $\nabla = \partial_x\,\mathbf{i} + \partial_y\,\mathbf{j} + \partial_z\,\mathbf{k}$, which of the following is (are) NOT defined?

I. $\nabla(\nabla\cdot\mathbf{F})$

II. $\nabla\times(\nabla\times\mathbf{F})$

III. $\nabla\cdot(\nabla\times\mathbf{F})$

IV. $\nabla\times(\nabla\cdot\mathbf{F})$

A. I only

B. II only

C. III only

D. $\boxed{\text{IV only}}$

E. III and IV

15. Evaluate

$$\oint_C (2y+x)dx + (4x-y^2)dy$$

where C is the boundary of the triangle with vertices $(0,0)$, $(1,1)$ and $(2,1)$ oriented counter clockwise.

A. $\boxed{1}$

B. 2

C. 3

D. 6

E. $\sqrt{2}$

16. Find the work done by

$$\mathbf{F}(x,y,z) = (2x+yz)\mathbf{i} + (-3y^2+xz)\mathbf{j} + (\frac{1}{z}+xy)\mathbf{k}$$

over the curve

$$\mathbf{r}(t) = t\mathbf{i} + 2t^2\mathbf{j} + \mathbf{k}$$

from the point $(0,0,1)$ to point $(1,2,1)$.

A. $\boxed{-5}$

B. 2

C. 3

D. 6

E. $\sqrt{2}$

17. Find the flux of

$$\mathbf{F} = x\mathbf{i} + y\mathbf{j} + (3 - 2z)\mathbf{k}$$

upward through the surface S which is the part of the paraboloid $z = 1 - x^2 - y^2$ lying above the plane $z = 0$.

A. -2π

B. 0

C. $\boxed{3\pi}$

D. 4π

E. 15π

18. Let S be the surface (lateral, top, and bottom) of the cylinder

$$x^2 + y^2 \leq 5, \quad 0 \leq z \leq 3.$$

Calculate the outward flux of the vector field

$$F(x, y, z) = y\mathbf{i} + x\mathbf{j} + z^2\mathbf{k}$$

through S.

A. 6π

B. 15π

C. 22π

D. $\boxed{45\pi}$

E. 90π

19. Let $\mathbf{F}(x, y, z) = -y\sin(z)\mathbf{i} + x\sin(z)\mathbf{j} + xy\cos(z)\mathbf{k}$, evaluate

$$\oint_C \mathbf{F} \cdot d\mathbf{r}$$

where C is the circle cut from the sphere $x^2 + y^2 + z^2 = 5$ by the plane $z = -1$, counter clockwise as viewed from above.

A. $\boxed{-8\pi\sin(1)}$

B. $-8\pi\cos(1)$

C. $8\pi\cos(3)$

D. 15π

E. 6π

20. Compute the area of the surface defined by parametric equations

$$x = u + v,\ y = u - v,\ z = 1 - u$$

where $0 \le u \le 1$ and $0 \le v \le u$.

A. $\frac{1}{3}$

B. $\sqrt{6}/3$

C. $\boxed{\sqrt{6}/2}$

D. $\sqrt{5}$

E. $\sqrt{6}$