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reducedForms::usage =
"reducedForms[d] produces a list of \n
  primitive reduced forms of discriminant -d, \n
  where d is a positive integer < 1.6*10^(19)\n
  and congruent to 0 or -1 (mod 4)";

formReduce::usage =
" formReduce[{a,b,c}] returns the\n
  unique reduced form properly equivalent to\n
  the positive definite binary form\n
  {a,b,c} = a x^2 + b x y + c y^2." ;

(* %%%%%%%%%%% *)

reducedForms[d_] :=
Module[{q = {}, m = Mod[-d, 4], s, a, b, c},

(* error check *)
Which[!(IntegerQ[d] && 0 < d < 16 * 10^(18)),
  Message[reducedForms::"bad argument"],
  Mod[d, 4] == 1 || Mod[d, 4] == 2,
  Print[
    "The discriminant -d should be an integer\n
    congruent to 0 or 1 (mod 4)."],
  True,

(* parameter okay--go ahead *)

  Do[
    b = Range[m, a, 2];
    If[b == {}, Continue[]];

(* Following line eliminates non-primitive forms and those with a>c *)
    b = Select[b,
      (#^2 + d >= 4 a^2 && GCD[a, #, (#^2 + d) / (4 a)] == 1 &);

    ( q = Join[q,
      { {a, Abs[#], (#^2 + d) / (4 a)},
        {a, -Abs[#], (#^2 + d) / (4 a)} }
      ];
      If[
        Abs[#] == a
      ||
        #^2 + d == 4 a^2
      ||
        # == 0,
        q = Drop[q, -1]
      ]
    ) & /@ b,
    {a, Sqrt[Abs[d] / 3]}

  ]
];

q]

(* ----- *)

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formReduce[{a_Integer, b_Integer, c_Integer}] :=
  Module[{q, red = {a, b, c}},

    (* error check *)
    If[b^2 >= 4 a c,
      Print["discriminant must be < 0."],
    (* Might want to add further error-checking *)

      Which[-a < b <= a && (a > c || (a == c && b < 0)),
        red = formReduce[{c, -b, a}],
        b > a || b <= -a,
          q = Quotient[b, 2 a] +
            Floor[Mod[b, 2 a] / (a + .5)];
          red = formReduce[
            {a, b - 2 a q, q^2 a - b q + c}],
        True,
          Return[red]
      ]
    ]
  ]

Do[If[Mod[d, 4] == 0 || Mod[d, 4] == 3, rd := reducedForms[d];
  Print["discrim = ", -d, ": ", "ClassNo = ", Length[rd], " ", rd]], {d, 1, 249}]
discrim = -3: ClassNo = 1 {{1, 1, 1}}
discrim = -4: ClassNo = 1 {{1, 0, 1}}
discrim = -7: ClassNo = 1 {{1, 1, 2}}
discrim = -8: ClassNo = 1 {{1, 0, 2}}
discrim = -11: ClassNo = 1 {{1, 1, 3}}
discrim = -12: ClassNo = 1 {{1, 0, 3}}
discrim = -15: ClassNo = 2 {{1, 1, 4}, {2, 1, 2}}
discrim = -16: ClassNo = 1 {{1, 0, 4}}
discrim = -19: ClassNo = 1 {{1, 1, 5}}
discrim = -20: ClassNo = 2 {{1, 0, 5}, {2, 2, 3}}
discrim = -23: ClassNo = 3 {{1, 1, 6}, {2, 1, 3}, {2, -1, 3}}
discrim = -24: ClassNo = 2 {{1, 0, 6}, {2, 0, 3}}
discrim = -27: ClassNo = 1 {{1, 1, 7}}
discrim = -28: ClassNo = 1 {{1, 0, 7}}
discrim = -31: ClassNo = 3 {{1, 1, 8}, {2, 1, 4}, {2, -1, 4}}
discrim = -32: ClassNo = 2 {{1, 0, 8}, {3, 2, 3}}
discrim = -35: ClassNo = 2 {{1, 1, 9}, {3, 1, 3}}
discrim = -36: ClassNo = 2 {{1, 0, 9}, {2, 2, 5}}
discrim = -39: ClassNo = 4 {{1, 1, 10}, {2, 1, 5}, {2, -1, 5}, {3, 3, 4}}

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discrim = -40: ClassNo = 2 {{1, 0, 10}, {2, 0, 5}}
discrim = -43: ClassNo = 1 {{1, 1, 11}}
discrim = -44: ClassNo = 3 {{1, 0, 11}, {3, 2, 4}, {3, -2, 4}}
discrim = -47: ClassNo = 5 {{1, 1, 12}, {2, 1, 6}, {2, -1, 6}, {3, 1, 4}, {3, -1, 4}}
discrim = -48: ClassNo = 2 {{1, 0, 12}, {3, 0, 4}}
discrim = -51: ClassNo = 2 {{1, 1, 13}, {3, 3, 5}}
discrim = -52: ClassNo = 2 {{1, 0, 13}, {2, 2, 7}}
discrim = -55: ClassNo = 4 {{1, 1, 14}, {2, 1, 7}, {2, -1, 7}, {4, 3, 4}}
discrim = -56: ClassNo = 4 {{1, 0, 14}, {2, 0, 7}, {3, 2, 5}, {3, -2, 5}}
discrim = -59: ClassNo = 3 {{1, 1, 15}, {3, 1, 5}, {3, -1, 5}}
discrim = -60: ClassNo = 2 {{1, 0, 15}, {3, 0, 5}}
discrim = -63: ClassNo = 4 {{1, 1, 16}, {2, 1, 8}, {2, -1, 8}, {4, 1, 4}}
discrim = -64: ClassNo = 2 {{1, 0, 16}, {4, 4, 5}}
discrim = -67: ClassNo = 1 {{1, 1, 17}}
discrim = -68: ClassNo = 4 {{1, 0, 17}, {2, 2, 9}, {3, 2, 6}, {3, -2, 6}}
discrim = -71: ClassNo = 7
  {{1, 1, 18}, {2, 1, 9}, {2, -1, 9}, {3, 1, 6}, {3, -1, 6}, {4, 3, 5}, {4, -3, 5}}
discrim = -72: ClassNo = 2 {{1, 0, 18}, {2, 0, 9}}
discrim = -75: ClassNo = 2 {{1, 1, 19}, {3, 3, 7}}
discrim = -76: ClassNo = 3 {{1, 0, 19}, {4, 2, 5}, {4, -2, 5}}
discrim = -79: ClassNo = 5 {{1, 1, 20}, {2, 1, 10}, {2, -1, 10}, {4, 1, 5}, {4, -1, 5}}
discrim = -80: ClassNo = 4 {{1, 0, 20}, {3, 2, 7}, {3, -2, 7}, {4, 0, 5}}
discrim = -83: ClassNo = 3 {{1, 1, 21}, {3, 1, 7}, {3, -1, 7}}
discrim = -84: ClassNo = 4 {{1, 0, 21}, {2, 2, 11}, {3, 0, 7}, {5, 4, 5}}
discrim = -87: ClassNo = 6
  {{1, 1, 22}, {2, 1, 11}, {2, -1, 11}, {3, 3, 8}, {4, 3, 6}, {4, -3, 6}}
discrim = -88: ClassNo = 2 {{1, 0, 22}, {2, 0, 11}}
discrim = -91: ClassNo = 2 {{1, 1, 23}, {5, 3, 5}}
discrim = -92: ClassNo = 3 {{1, 0, 23}, {3, 2, 8}, {3, -2, 8}}
discrim = -95: ClassNo = 8
  {{1, 1, 24}, {2, 1, 12}, {2, -1, 12}, {3, 1, 8}, {3, -1, 8}, {4, 1, 6}, {4, -1, 6}, {5, 5, 6}}
discrim = -96: ClassNo = 4 {{1, 0, 24}, {3, 0, 8}, {4, 4, 7}, {5, 2, 5}}
discrim = -99: ClassNo = 2 {{1, 1, 25}, {5, 1, 5}}
discrim = -100: ClassNo = 2 {{1, 0, 25}, {2, 2, 13}}
discrim = -103: ClassNo = 5 {{1, 1, 26}, {2, 1, 13}, {2, -1, 13}, {4, 3, 7}, {4, -3, 7}}
discrim = -104: ClassNo = 6
  {{1, 0, 26}, {2, 0, 13}, {3, 2, 9}, {3, -2, 9}, {5, 4, 6}, {5, -4, 6}}
discrim = -107: ClassNo = 3 {{1, 1, 27}, {3, 1, 9}, {3, -1, 9}}

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discrim = -108: ClassNo = 3  {{1, 0, 27}, {4, 2, 7}, {4, -2, 7}}
discrim = -111: ClassNo = 8
  {{1, 1, 28}, {2, 1, 14}, {2, -1, 14}, {3, 3, 10}, {4, 1, 7}, {4, -1, 7}, {5, 3, 6}, {5, -3, 6}}
discrim = -112: ClassNo = 2  {{1, 0, 28}, {4, 0, 7}}
discrim = -115: ClassNo = 2  {{1, 1, 29}, {5, 5, 7}}
discrim = -116: ClassNo = 6
  {{1, 0, 29}, {2, 2, 15}, {3, 2, 10}, {3, -2, 10}, {5, 2, 6}, {5, -2, 6}}
discrim = -119: ClassNo = 10  {{1, 1, 30}, {2, 1, 15}, {2, -1, 15},
  {3, 1, 10}, {3, -1, 10}, {4, 3, 8}, {4, -3, 8}, {5, 1, 6}, {5, -1, 6}, {6, 5, 6}}
discrim = -120: ClassNo = 4  {{1, 0, 30}, {2, 0, 15}, {3, 0, 10}, {5, 0, 6}}
discrim = -123: ClassNo = 2  {{1, 1, 31}, {3, 3, 11}}
discrim = -124: ClassNo = 3  {{1, 0, 31}, {5, 4, 7}, {5, -4, 7}}
discrim = -127: ClassNo = 5  {{1, 1, 32}, {2, 1, 16}, {2, -1, 16}, {4, 1, 8}, {4, -1, 8}}
discrim = -128: ClassNo = 4  {{1, 0, 32}, {3, 2, 11}, {3, -2, 11}, {4, 4, 9}}
discrim = -131: ClassNo = 5  {{1, 1, 33}, {3, 1, 11}, {3, -1, 11}, {5, 3, 7}, {5, -3, 7}}
discrim = -132: ClassNo = 4  {{1, 0, 33}, {2, 2, 17}, {3, 0, 11}, {6, 6, 7}}
discrim = -135: ClassNo = 6
  {{1, 1, 34}, {2, 1, 17}, {2, -1, 17}, {4, 3, 9}, {4, -3, 9}, {5, 5, 8}}
discrim = -136: ClassNo = 4  {{1, 0, 34}, {2, 0, 17}, {5, 2, 7}, {5, -2, 7}}
discrim = -139: ClassNo = 3  {{1, 1, 35}, {5, 1, 7}, {5, -1, 7}}
discrim = -140: ClassNo = 6
  {{1, 0, 35}, {3, 2, 12}, {3, -2, 12}, {4, 2, 9}, {4, -2, 9}, {5, 0, 7}}
discrim = -143: ClassNo = 10  {{1, 1, 36}, {2, 1, 18}, {2, -1, 18},
  {3, 1, 12}, {3, -1, 12}, {4, 1, 9}, {4, -1, 9}, {6, 1, 6}, {6, 5, 7}, {6, -5, 7}}
discrim = -144: ClassNo = 4  {{1, 0, 36}, {4, 0, 9}, {5, 4, 8}, {5, -4, 8}}
discrim = -147: ClassNo = 2  {{1, 1, 37}, {3, 3, 13}}
discrim = -148: ClassNo = 2  {{1, 0, 37}, {2, 2, 19}}
discrim = -151: ClassNo = 7
  {{1, 1, 38}, {2, 1, 19}, {2, -1, 19}, {4, 3, 10}, {4, -3, 10}, {5, 3, 8}, {5, -3, 8}}
discrim = -152: ClassNo = 6
  {{1, 0, 38}, {2, 0, 19}, {3, 2, 13}, {3, -2, 13}, {6, 4, 7}, {6, -4, 7}}
discrim = -155: ClassNo = 4  {{1, 1, 39}, {3, 1, 13}, {3, -1, 13}, {5, 5, 9}}
discrim = -156: ClassNo = 4  {{1, 0, 39}, {3, 0, 13}, {5, 2, 8}, {5, -2, 8}}
discrim = -159: ClassNo = 10  {{1, 1, 40}, {2, 1, 20}, {2, -1, 20},
  {3, 3, 14}, {4, 1, 10}, {4, -1, 10}, {5, 1, 8}, {5, -1, 8}, {6, 3, 7}, {6, -3, 7}}
discrim = -160: ClassNo = 4  {{1, 0, 40}, {4, 4, 11}, {5, 0, 8}, {7, 6, 7}}
discrim = -163: ClassNo = 1  {{1, 1, 41}}
discrim = -164: ClassNo = 8
  {{1, 0, 41}, {2, 2, 21}, {3, 2, 14}, {3, -2, 14}, {5, 4, 9}, {5, -4, 9}, {6, 2, 7}, {6, -2, 7}}
discrim = -167: ClassNo = 11  {{1, 1, 42}, {2, 1, 21}, {2, -1, 21}, {3, 1, 14},
  {3, -1, 14}, {4, 3, 11}, {4, -3, 11}, {6, 1, 7}, {6, -1, 7}, {6, 5, 8}, {6, -5, 8}}
discrim = -168: ClassNo = 4  {{1, 0, 42}, {2, 0, 21}, {3, 0, 14}, {6, 0, 7}}
discrim = -171: ClassNo = 4  {{1, 1, 43}, {5, 3, 9}, {5, -3, 9}, {7, 5, 7}}

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discrim = -172: ClassNo = 3  {{1, 0, 43}, {4, 2, 11}, {4, -2, 11}}
discrim = -175: ClassNo = 6
  {{1, 1, 44}, {2, 1, 22}, {2, -1, 22}, {4, 1, 11}, {4, -1, 11}, {7, 7, 8}}
discrim = -176: ClassNo = 6
  {{1, 0, 44}, {3, 2, 15}, {3, -2, 15}, {4, 0, 11}, {5, 2, 9}, {5, -2, 9}}
discrim = -179: ClassNo = 5  {{1, 1, 45}, {3, 1, 15}, {3, -1, 15}, {5, 1, 9}, {5, -1, 9}}
discrim = -180: ClassNo = 4  {{1, 0, 45}, {2, 2, 23}, {5, 0, 9}, {7, 4, 7}}
discrim = -183: ClassNo = 8
  {{1, 1, 46}, {2, 1, 23}, {2, -1, 23}, {3, 3, 16}, {4, 3, 12}, {4, -3, 12}, {6, 3, 8}, {6, -3, 8}}
discrim = -184: ClassNo = 4  {{1, 0, 46}, {2, 0, 23}, {5, 4, 10}, {5, -4, 10}}
discrim = -187: ClassNo = 2  {{1, 1, 47}, {7, 3, 7}}
discrim = -188: ClassNo = 5  {{1, 0, 47}, {3, 2, 16}, {3, -2, 16}, {7, 6, 8}, {7, -6, 8}}
discrim = -191: ClassNo = 13 {{1, 1, 48}, {2, 1, 24}, {2, -1, 24}, {3, 1, 16}, {3, -1, 16},
  {4, 1, 12}, {4, -1, 12}, {5, 3, 10}, {5, -3, 10}, {6, 1, 8}, {6, -1, 8}, {6, 5, 9}, {6, -5, 9}}
discrim = -192: ClassNo = 4  {{1, 0, 48}, {3, 0, 16}, {4, 4, 13}, {7, 2, 7}}
discrim = -195: ClassNo = 4  {{1, 1, 49}, {3, 3, 17}, {5, 5, 11}, {7, 1, 7}}
discrim = -196: ClassNo = 4  {{1, 0, 49}, {2, 2, 25}, {5, 2, 10}, {5, -2, 10}}
discrim = -199: ClassNo = 9  {{1, 1, 50}, {2, 1, 25}, {2, -1, 25},
  {4, 3, 13}, {4, -3, 13}, {5, 1, 10}, {5, -1, 10}, {7, 5, 8}, {7, -5, 8}}
discrim = -200: ClassNo = 6
  {{1, 0, 50}, {2, 0, 25}, {3, 2, 17}, {3, -2, 17}, {6, 4, 9}, {6, -4, 9}}
discrim = -203: ClassNo = 4  {{1, 1, 51}, {3, 1, 17}, {3, -1, 17}, {7, 7, 9}}
discrim = -204: ClassNo = 6
  {{1, 0, 51}, {3, 0, 17}, {4, 2, 13}, {4, -2, 13}, {5, 4, 11}, {5, -4, 11}}
discrim = -207: ClassNo = 6
  {{1, 1, 52}, {2, 1, 26}, {2, -1, 26}, {4, 1, 13}, {4, -1, 13}, {8, 7, 8}}
discrim = -208: ClassNo = 4  {{1, 0, 52}, {4, 0, 13}, {7, 4, 8}, {7, -4, 8}}
discrim = -211: ClassNo = 3  {{1, 1, 53}, {5, 3, 11}, {5, -3, 11}}
discrim = -212: ClassNo = 6
  {{1, 0, 53}, {2, 2, 27}, {3, 2, 18}, {3, -2, 18}, {6, 2, 9}, {6, -2, 9}}
discrim = -215: ClassNo = 14
  {{1, 1, 54}, {2, 1, 27}, {2, -1, 27}, {3, 1, 18}, {3, -1, 18}, {4, 3, 14}, {4, -3, 14},
  {5, 5, 12}, {6, 1, 9}, {6, -1, 9}, {6, 5, 10}, {6, -5, 10}, {7, 3, 8}, {7, -3, 8}}
discrim = -216: ClassNo = 6
  {{1, 0, 54}, {2, 0, 27}, {5, 2, 11}, {5, -2, 11}, {7, 6, 9}, {7, -6, 9}}
discrim = -219: ClassNo = 4  {{1, 1, 55}, {3, 3, 19}, {5, 1, 11}, {5, -1, 11}}
discrim = -220: ClassNo = 4  {{1, 0, 55}, {5, 0, 11}, {7, 2, 8}, {7, -2, 8}}
discrim = -223: ClassNo = 7
  {{1, 1, 56}, {2, 1, 28}, {2, -1, 28}, {4, 1, 14}, {4, -1, 14}, {7, 1, 8}, {7, -1, 8}}
discrim = -224: ClassNo = 8
  {{1, 0, 56}, {3, 2, 19}, {3, -2, 19}, {4, 4, 15}, {5, 4, 12}, {5, -4, 12}, {7, 0, 8}, {8, 8, 9}}
discrim = -227: ClassNo = 5  {{1, 1, 57}, {3, 1, 19}, {3, -1, 19}, {7, 5, 9}, {7, -5, 9}}
discrim = -228: ClassNo = 4  {{1, 0, 57}, {2, 2, 29}, {3, 0, 19}, {6, 6, 11}}
discrim = -231: ClassNo = 12 {{1, 1, 58}, {2, 1, 29}, {2, -1, 29}, {3, 3, 20}, {4, 3, 15},
  {4, -3, 15}, {5, 3, 12}, {5, -3, 12}, {6, 3, 10}, {6, -3, 10}, {7, 7, 10}, {8, 5, 8}}

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discrim = -232: ClassNo = 2  {{1, 0, 58}, {2, 0, 29}}
discrim = -235: ClassNo = 2  {{1, 1, 59}, {5, 5, 13}}
discrim = -236: ClassNo = 9  {{1, 0, 59}, {3, 2, 20}, {3, -2, 20},
  {4, 2, 15}, {4, -2, 15}, {5, 2, 12}, {5, -2, 12}, {7, 4, 9}, {7, -4, 9}}
discrim = -239: ClassNo = 15 {{1, 1, 60}, {2, 1, 30}, {2, -1, 30},
  {3, 1, 20}, {3, -1, 20}, {4, 1, 15}, {4, -1, 15}, {5, 1, 12}, {5, -1, 12},
  {6, 1, 10}, {6, -1, 10}, {6, 5, 11}, {6, -5, 11}, {8, 7, 9}, {8, -7, 9}}
discrim = -240: ClassNo = 4  {{1, 0, 60}, {3, 0, 20}, {4, 0, 15}, {5, 0, 12}}
discrim = -243: ClassNo = 3  {{1, 1, 61}, {7, 3, 9}, {7, -3, 9}}
discrim = -244: ClassNo = 6
  {{1, 0, 61}, {2, 2, 31}, {5, 4, 13}, {5, -4, 13}, {7, 6, 10}, {7, -6, 10}}
discrim = -247: ClassNo = 6
  {{1, 1, 62}, {2, 1, 31}, {2, -1, 31}, {4, 3, 16}, {4, -3, 16}, {8, 3, 8}}
discrim = -248: ClassNo = 8
  {{1, 0, 62}, {2, 0, 31}, {3, 2, 21}, {3, -2, 21}, {6, 4, 11}, {6, -4, 11}, {7, 2, 9}, {7, -2, 9}}
Length[reducedForms[77777711]] // Timing
{105.468, 10970}

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