Kiril Datchev
MA 546
Spring 2025

Homework 1

Due January 15th on paper at the beginning of class. Please let me know if you have a question or find a mistake.

- Do Problems 7, 17, and 27 from Chapter I of Reed and Simon's *Functional Analysis*. (These are the only three problems that should be handed in.)
- For those who want to really brush up: We will assume general knowledge of Chapter I, although it is reasonable to have gaps here, especially in the measure theory which gets quite technical. More complicated results will be reviewed as they are needed, but it will help to study carefully at least some of the theorems and examples, and work through more problems. As always, watch out for typos, mistakes, and sudden jumps in difficulty. I particularly recommend:
  - Section I.2: Example 3, the construction of the Riemann integral on the space PC, problems 8 and 9.
  - Section I.4: Examples 1, 2, and 3, problem 25, and also practice using the monotone convergence theorem to prove the dominated convergence theorem (hint below[1]).
  - All of Sections I.5 and I.6, especially problems 26, 29, and 32.

[1]Reduce to the real-valued case, and then sandwich f_n between two monotone sequences by writing $g_n \leq f_n \leq h_n$, where $g_n = \lim_{k\to\infty} \min(f_n, f_{n+1}, \dots, f_{n+k})$ and $h_n = \lim_{k\to\infty} \max(f_n, f_{n+1}, \dots, f_{n+k})$.