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MA 546
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Homework 2

Due January 22nd on paper at the beginning of class. Please let me know if you have a question or find a mistake. The book is Reed and Simon's *Functional Analysis*.

1. Use just Theorem II.1, Example 2 preceding it, and elementary integration, to show that

$$\lim_{N\to\infty} \int_0^\pi \left| \frac{\pi - t}{2} - \sum_{n=1}^{N} \frac{\sin nt}{n} \right|^2 dt = 0, \tag{1}$$

   if and only if

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}. \tag{2}$$

2. Do Problems 6 and 7 from Chapter II; for the uniqueness statement in the preceding lemma, use the weaker hypothesis (under which the proof of the lemma works the same) that $\mathscr{M}$ is a closed subset of $\mathscr{H}$, not necessarily a subspace, such that $\frac{v+w}{2} \in \mathscr{M}$ whenever v and w are in $\mathscr{M}$.
3. Give explicit formulas for the projections in Theorem II.3 in cases where
   (a) $\mathscr{H} = L^2(\mathbb{R}^d)$, $\mathscr{M} = \{f \in \mathscr{H} : f(x) = f(-x)\}$.
   (b) $\mathscr{H} = \mathbb{C}$, $\mathscr{M} = \mathbb{R}$.

   What other similar examples can you think of?