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MA 546
Spring 2025

Homework 3

Due January 29th on paper at the beginning of class. Please let me know if you have a question or find a mistake. The book is Reed and Simon's *Functional Analysis*.

1. Do Problems 8, 9, 10, and 27 from Chapter II.
2. Mimic the proof of Theorem II.5 to show that every vector space has a basis.
3. Let I and J be intervals in $\mathbb{R}$. Let $\{\varphi_m\}_{m\in\mathbb{N}}$ and $(\psi_n)_{n\in\mathbb{N}}$ be orthonormal bases of $L^2(I)$ and $L^2(J)$ respectively . For each $(x, y) \in I \times J$, let $\chi_{m,n}(x, y) = \varphi_m(x)\psi_n(y)$. Use Fubini's theorem to prove that $\{\chi_{m,n}\}_{(m,n)\in\mathbb{N}\times\mathbb{N}}$ is an orthonormal basis of $L^2(I \times J)$.[1]

[1] Section II.4 has a general theory of products of Hilbert spaces but this is not needed here. Actually the proof works the same if I and J are measurable subsets of Euclidean space, or manifolds, etc.