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MA 546
Spring 2025

Homework 5

Due February 19th on paper at the beginning of class. Please let me know if you have a question or find a mistake.

- Exercises 1 and 2 from `https://www.math.purdue.edu/~kdatchev/546/hb.pdf`.
- Let $I \subset \mathbb{R}$ be an interval, let X be the space of bounded continuous functions from I to $\mathbb{F}$, and for every $p \in I$ and $h \in \mathbb{R} \setminus \{0\}$ such that $p + h \in I$, define $T_{p,h} \in X^*$ by

$$T_{p,h} f = \frac{f(p+h) - f(p)}{h}.$$

  Prove that $\|T_{p,h}\|_{X^*} = 2/h$, and use the uniform boundedness principle[1] to show that, for any fixed $p \in I$, the set of f which are differentiable at p is meager.
- Problem 19 from Chapter III of *Reed and Simon*.

[1] Recall the statement that if $\mathcal{F}$ is a set of bounded linear maps from a normed vector space X to a normed vector space Y, and if $\sup_{T \in \mathcal{F}} \|T\|_{X \to Y} = +\infty$, then the set $\{f \in X \colon \sup_{T \in \mathcal{F}} \|Tf\|_Y < +\infty\}$ is meager.