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MA 546
Spring 2025

Homework 6

Due February 26th on paper at the beginning of class. Please let me know if you have a question or find a mistake.

- For $\Omega \subset \mathbb{R}^n$ open, bounded, and nonempty, and m a nonnegative integer, let $C_b^m(\Omega)$ be the set of all functions $u\colon \Omega \to \mathbb{F}$ such that u and all its partial derivatives up to order m are continuous and bounded.
  (a) Prove that $C_b^m(\Omega)$ is a Banach space under the norm $\|u\|_m = \max_{|\alpha|\le m} \sup_{x\in\Omega} |\partial^\alpha u(x)|$.
  (b) Mimic the proof in Example 2.4.3 of *Hörmander* to show if there is a partial differential operator with constant coefficients $P(D)$ of order $\le m$ such that every solution $u \in C_b^m(\Omega)$ to the equation $P(D)u = 0$ is in fact in $C_b^{m+1}(\Omega)$, then $n = 1$.
- Exercises 1) and 2) from `https://www.math.purdue.edu/~kdatchev/546/frechet.pdf`.