Kiril Datchev
MA 546
Spring 2025

Homework 8

Due March 12th on paper at the beginning of class. Please let me know if you have a question or find a mistake.

1. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be linear, with eigenvectors u and v such that $Tu = u$, $Tv = -v$, and $0 < \theta \leq \pi/2$, where θ is the angle between u and v. Let

$$d_n(\theta) = \|T^n\| - r(T)^n,$$

   where $\|\cdot\|$ is the operator norm and $r(\cdot)$ is the spectral radius. Give a simple formula for $d_n(\theta)$ in terms of the cotangent function, and sketch $d_n(\theta)$ as a function of θ for a few typical values of n.
2. Let $\mathcal{H}$ be a Hilbert space, and T a linear homeomorphism $\mathcal{H} \to \mathcal{H}$. Prove that $\sigma(T^{-1}) = \sigma(T)^{-1}$, i.e. that λ is in the spectrum of T if and only if λ^{-1} is in the spectrum of T^{-1}.
3. Exercise VII.3(a) (not VI.3(a)!) from Reed and Simon.
4. Exercises 4.4 and 4.5 from Borthwick.