Today: §1.1 & 1.2.

leading entry

$$\left[\begin{array}{cc|c} 1 & 1 & 8 \\ 2 & 4 & 22 \end{array}\right] \rightsquigarrow \left[\begin{array}{cc|c} 1 & 1 & 8 \\ 0 & 2 & 6 \end{array}\right]$$

row echelon form REF

$$\begin{cases} p+d=8 \\ 2d=6 \end{cases}$$

reduced row echelon form RREF

$$\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 3 \end{array}\right] \leftarrow \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 2 & 6 \end{array}\right]$$

Def: Leading entry is the first nonzero entry in a row.

Row echelon form: The entries below all leading entries are zeros & leading entries proceed rightward.

Reduced row echelon form: Leading entries are 1 and all entries above and below leading entries are 0.

Main example

$$\begin{cases} 3x_2-6x_3+6x_4+4x_5=-5 \\ 3x_1-7x_2+8x_3-5x_4+8x_5=9 \\ 3x_1-9x_2+12x_3-9x_4+6x_5=15 \end{cases}$$

$$\begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \left[\begin{array}{ccccc|c} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{array}\right] \end{array}$$

3, -7, 4 are pivot entries

$R_1 \leftrightarrow R_3$. interchange

$$\left[\begin{array}{ccccc|c} 3 & -9 & 12 & -9 & 6 & 15 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 0 & 3 & -6 & 6 & 4 & -5 \end{array}\right] \xrightarrow{R_2 \mapsto R_2 - R_1} \left[\begin{array}{ccccc|c} 3 & -9 & 12 & -9 & 6 & 15 \\ 0 & 2 & -4 & 4 & 2 & -6 \\ 0 & 3 & -6 & 6 & 4 & -5 \end{array}\right]$$

$\big\downarrow R_3 \mapsto R_3 - \frac{3}{2} R_2$

$$\left[\begin{array}{ccccc|c} 3 & -9 & 12 & -9 & 6 & 15 \\ 0 & 2 & -4 & 4 & 2 & -6 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

row echelon form

$x_5 = 4$

Can solve for x_2 in terms of x_3 & x_4

& for x_1 in terms of x_2, x_3, & x_4

x_3 and x_4 can be chosen at random.

There is a solution for any choice of x_3 & x_4.

$\big\downarrow R_1 \mapsto \frac{1}{3} R_1$

$$\left[\begin{array}{ccccc|c} 1 & -3 & 4 & -3 & 2 & 5 \\ 0 & 2 & -4 & 4 & 2 & -6 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

$\xrightarrow{R_2 \mapsto \frac{1}{2} R_2}$

$$\left[\begin{array}{ccccc|c} 1 & -3 & 4 & -3 & 2 & 5 \\ 0 & 1 & -2 & 2 & 1 & -3 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

$\big\downarrow R_1 \mapsto R_1 + 3R_2$

$$\left[\begin{array}{ccccc|c} 1 & 0 & -2 & 3 & 5 & -4 \\ 0 & 1 & -2 & 2 & 1 & -3 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right] \xrightarrow{R_2 \mapsto R_2 - R_3} \left[\begin{array}{ccccc|c} 1 & 0 & -2 & 3 & 5 & -4 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

$\big\downarrow R_1 \mapsto R_1 - 5R_3$

$$\left[\begin{array}{ccccc|c} 1 & 0 & -2 & 3 & 0 & -24 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

RREF

pivot columns

$$\begin{cases} x_1 - 2x_3 + 3x_4 = -24 \\ x_2 - 2x_3 + 2x_4 = -7 \\ x_5 = 4 \end{cases}$$

$$\begin{cases} x_1 = 2x_3 - 3x_4 - 24 \\ x_2 = 2x_3 - 2x_4 - 7 \\ x_5 = 4 \end{cases}$$

solution.

$$\begin{cases} x_1 = 2s - 3t - 24 \\ x_2 = 2s - 2t - 7 \\ x_3 = s \\ x_4 = t \\ x_5 = 4 \end{cases}$$

solution in parametric form.