Today: §1.2

pivot positions

$$\left[\begin{array}{ccc|c} 0 & 1 & -4 & 8 \\ 2 & -3 & 2 & 1 \\ 4 & -8 & 12 & 1 \end{array}\right] \qquad \begin{cases} x_2 - 4x_3 = 8 \\ 2x_1 - 3x_2 + 2x_3 = 1 \\ 4x_1 - 8x_2 + 12x_3 = 1 \end{cases}$$

$R_1 \leftrightarrow R_2$

pivot

$$\left[\begin{array}{ccc|c} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 4 & -8 & 12 & 1 \end{array}\right] \xrightarrow{R_3 \mapsto R_3 - 2R_1} \left[\begin{array}{ccc|c} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & -2 & 8 & -1 \end{array}\right]$$

pivot

$R_3 \mapsto R_3 + 2R_2$

$$\left[\begin{array}{ccc|c} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & 15 \end{array}\right] \quad \text{REF}$$

$$\begin{cases} 2x_1 - 3x_2 + 2x_3 = 1 \\ x_2 - 4x_3 = 8 \\ \boxed{0 = 15} \end{cases}$$

This system of linear, and the equivalent original system, is <u>inconsistent</u>.

$R_1 \mapsto \frac{1}{2} R_1$

$$\left[\begin{array}{ccc|c} 1 & -3/2 & 1 & 1/2 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & 15 \end{array}\right]$$

$R_1 + \frac{3}{2} R_2$

pivots

$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & 25/2 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & 15 \end{array}\right]$$

pivot columns

REF:

$$\left[\begin{array}{ccccc|c} 0 & 1 & 3 & -1 & 2 & 13 \\ 0 & 0 & 0 & 4 & -8 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

This corresponds to a <u>consistent</u> system of linear eq'ns

Solve for pivot variables ($\leftrightarrow$ pivot columns) $(x_2, x_4)$ in terms of subsequent variables. $x_1, x_3, x_5$ are free.

Free variables $\leftrightarrow$ columns w/o a pivot.

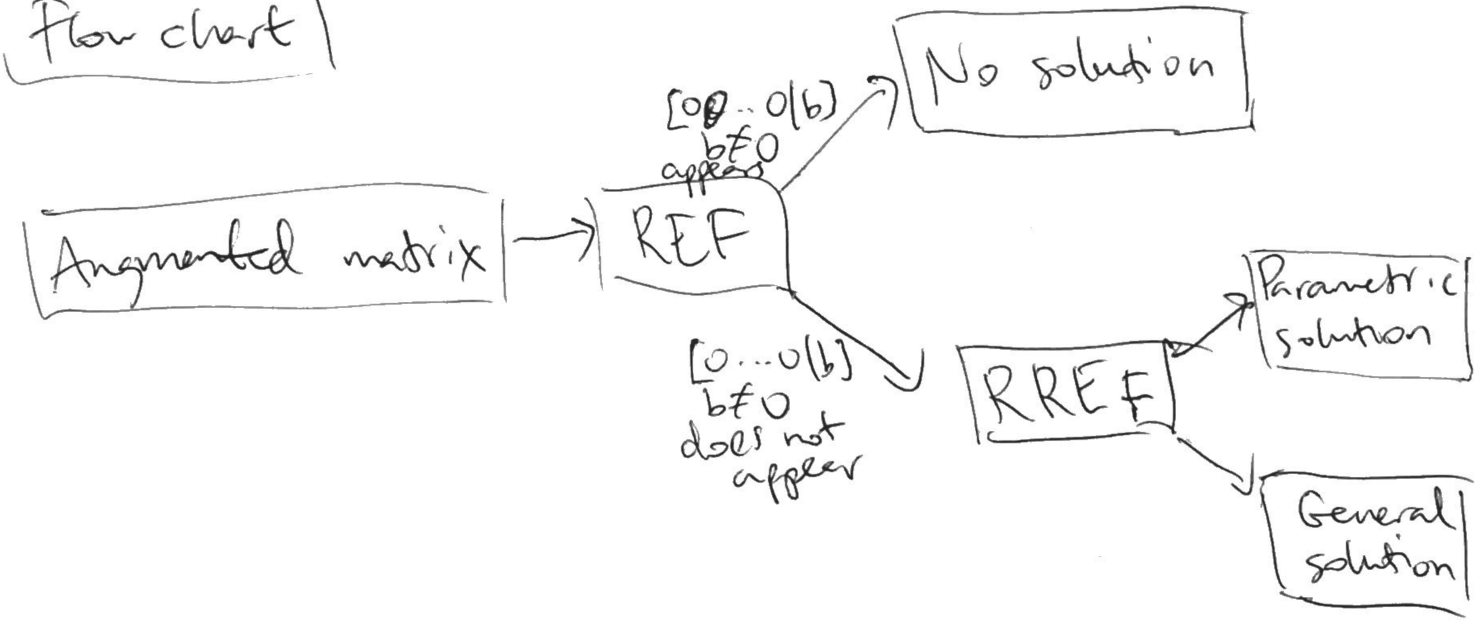
Thm: A system of linear eqns is inconsistent if and only if ~~the~~ a REF of the associated augmented matrix contains a row of the form $[0 \ 0 \ \dots \ 0 \mid b]$, $b \neq 0$.

Pf: If $[0 \dots 0 \mid b]$, $b \neq 0$ appears, the system is inconsistent.

Otherwise, can choose any values for free variables and solve for pivot variables in terms of subsequent variables.

Get a value for the last variable and work backwards.

Flow chart



Linear algebra

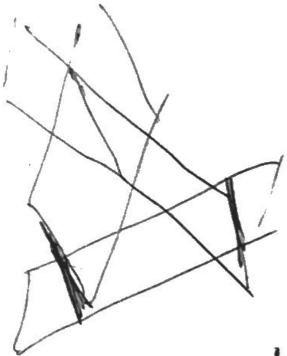
Geometry

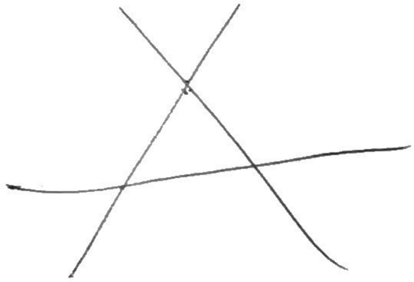
## Geometry:

$$\begin{cases} x_2 - 4x_3 = 8 \\ 2x_1 - 3x_2 + 2x_3 = 1 \\ 4x_1 - 8x_2 + 12x_3 = 1 \end{cases}$$

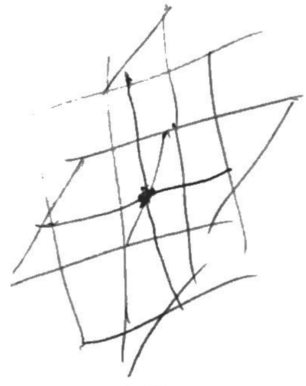
$x_3 = 0$

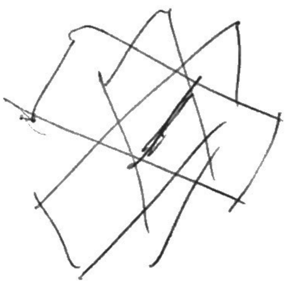
$x_1, x_2$-plane





inconsistent system





$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \quad \begin{array}{l} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{array}$$

consistent with one solution

When do you have only one solution?

Exactly when there are no free variables & system is consistent.

every column has a pivot.

## 2 eqns in 2 variables

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 1 & m & b \end{array}\right] \xrightarrow{R_2 \mapsto R_2 - R_1} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & m-2 & b-1 \end{array}\right] \text{ REF}$$

If $m = 2$, $b = 1$, the solution set is a line.

If $m = 2$, $b \neq 1$, system is inconsistent.

If $m \neq 2$, there is a unique solution.