§2.1 Matrix operations

Last time: Every linear transformation is a matrix transformation

$T(\vec{x}) = A\vec{x}$

← standard matrix for T

Today: Ways to combine matrices to get new ones.

Eg: $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 & 3 \\ 1 & -1 & -3 \end{bmatrix}$

$T(\vec{x}) = A\vec{x}$ $\quad$ $S(\vec{x}) = A\vec{x} + B\vec{x}$

$T: \mathbb{R}^3 \to \mathbb{R}^2$ $\quad$ $S: \mathbb{R}^3 \to \mathbb{R}^2$

S is a linear transformation

What is its standard matrix?

$$S(\vec{x}) = \left[S(\vec{e}_1) \;\; S(\vec{e}_2) \;\; S(\vec{e}_3) \right] \vec{x}$$

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & 5 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & -1 & 3 \\ 1 & -1 & -3 \end{bmatrix}$$

$$S(\vec{e}_1) = A\vec{e}_1 + B\vec{e}_1$$

$$= \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$S(\vec{e}_2) = \begin{bmatrix} 2 \\ 4 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$S(\vec{e}_3) = \begin{bmatrix} -1 \\ 5 \end{bmatrix} + \begin{bmatrix} 3 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$S(\vec{x}) = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 3 & 2 \end{bmatrix} \vec{x}$$

standard matrix

Call it A+B

<u>Matrix addition</u>: If A, B are $m \times n$ matrices, $A+B$ is the $m \times n$ matrix obtained by entrywise addition.

$$\underset{A}{\begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & 5 \end{bmatrix}} + \underset{B}{\begin{bmatrix} 0 & -1 & 3 \\ 1 & -1 & -3 \end{bmatrix}} = \underset{A+B}{\begin{bmatrix} 1 & 1 & 2 \\ 3 & 3 & 2 \end{bmatrix}}$$

Eg: $U(\vec{x}) = 3(A\vec{x})$ $\quad A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & 5 \end{bmatrix}$

What is the standard matrix of U?

$$U(\vec{x}) = \begin{bmatrix} U(\vec{e}_1) & U(\vec{e}_2) & U(\vec{e}_3) \end{bmatrix}$$

$$= \begin{bmatrix} 3(A\vec{e}_1) & 3(A\vec{e}_2) & 3(A\vec{e}_3) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 6 & -3 \\ 6 & 12 & 15 \end{bmatrix}$$

Call this $3A$.

$$3\begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & -3 \\ 6 & 12 & 15 \end{bmatrix}$$

$$(A+B)\vec{x} = A\vec{x} + B\vec{x}$$

$$(cA)\vec{x} = c(A\vec{x})$$

$$\begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix} = 0 \quad m \times n \text{ matrix}$$

Often just write 0 and size of matrix is clear from context.

$$A + 0 = A = 0 + A$$

$$0A = 0$$

scaler matrix

Eg: $V(\vec{x}) = C(D\vec{x})$ $\quad C = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$

$D = \begin{bmatrix} 0 & -1 & 3 \\ 1 & -1 & -3 \end{bmatrix}$

$V : \mathbb{R}^3 \to \mathbb{R}^2$ is a linear transformation

What is its standard matrix?

$$\begin{bmatrix} V(\vec{e}_1) & V(\vec{e}_2) & V(\vec{e}_3) \end{bmatrix}$$

$C = \begin{bmatrix} 1 & 2 \\ & 0 \end{bmatrix}$

$$= \begin{bmatrix} C(D\vec{e}_1) & C(D\vec{e}_2) & C(D\vec{e}_3) \end{bmatrix} \quad D = \begin{bmatrix} 0 & -1 & 3 \\ 1 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} C\begin{bmatrix} 0 \\ 1 \end{bmatrix} & C\begin{bmatrix} -1 \\ -1 \end{bmatrix} & C\begin{bmatrix} 3 \\ -3 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -3 \\ 0 & -1 & 3 \end{bmatrix}$$

Call this CD

$$CD = \begin{bmatrix} C\vec{d}_1 & C\vec{d}_2 & \cdots & C\vec{d}_p \end{bmatrix}$$

where $D = \begin{bmatrix} \vec{d}_1 & \cdots & \vec{d}_p \end{bmatrix}$

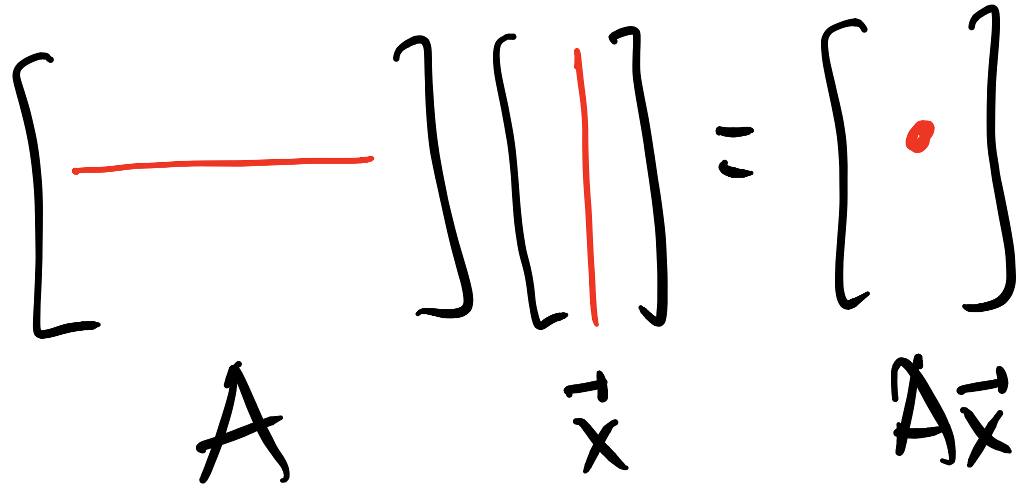
This only makes sense if #rows(D) = #columns(C)

$$(CD)\vec{x} = C(D\vec{x})$$

C $\qquad$ D $\qquad$ CD

$l \times m$ $\qquad$ $m \times n$ $\qquad$ $l \times n$

<u>Reminder</u>: $A\vec{x}$



<u>Now</u>: CD

$$\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 & 3 \\ 1 & -1 & -3 \end{bmatrix} = \begin{bmatrix} & & \\ & -1 & \end{bmatrix}$$

$$-1 = 1(-1) + 0(-1)$$

What is the standard matrix of $T(\vec{x}) = c\vec{x}$? $\quad T: \mathbb{R}^n \to \mathbb{R}^n$.

$$\begin{bmatrix} c & 0 & \cdots & 0 \\ 0 & c & & \vdots \\ \vdots & 0 & & \\ 0 & 0 & & c \end{bmatrix}$$

diagonal entries

main diagonal

Diagonal matrices are those whose only nonzero entries are diagonal entries.

Eg. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Transpose of a matrix

A^T

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}.$$

$$(A^T)^T = A$$

$$(AB)^T = B^T A^T.$$

$$(A+B)^T = A^T + B^T$$

$$(AB)C = A(BC)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$ RREF, but not diagonal