Homework 1/18

① Prove $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ if $b, d \neq 0$

② If $a < b$, $c < 0$, show that $ac > bc$

③ Whether $a \in \mathbb{P}$ or $-a \in \mathbb{P}$, prove $a \cdot a > 0$.

(Do not write a^2 for $a \cdot a$. This notation has not been introduced yet. Even notation "2" has not been introduced.)

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① Prove $2 \cdot 2 = 4$.

② If $p > 0$, prove that $1/p > 0$.

③ Given $a \in \mathbb{R}$, show that the equation

$$x \cdot x = a, \quad x \in \mathbb{R}, \text{ has}$$

(i) no solution if $a < 0$;

(ii) exactly one solution if $a = 0$;

(iii) at most two solutions if $a > 0$.

(We will later see that in case (iii) there are exactly two solutions. But this cannot be proved from the axioms we have at this point.)

HW 1-23

(1) If $x, y \in \mathbb{Q}$, then $x+y \in \mathbb{Q}$; $x-y \in \mathbb{Q}$; $xy \in \mathbb{Q}$; and if $y \neq 0$, then $x/y \in \mathbb{Q}$.

Choose one of these 4 statements and prove it.

(2) If $x \geq -1$ and $n \in \mathbb{N}$, prove $(1+x)^n \geq 1+nx$.

This is a famous inequality, but do not look it up in a book or on the web. Try to do it on your own.

Hint:

Try mathematical induction.