

HW 2-15

① Prove $\lim_{k \rightarrow \infty} \sqrt[k]{2} = 1$. Hint: \leq Bernoulli's

② Suppose a sequence (y_n) is defined recursively by

$$y_1 = 4; \quad y_{n+1} = \frac{1}{2} \left(y_n + \frac{3}{y_n} \right), \quad n \in \mathbb{N}.$$

Find (and, of course, prove) $\lim_{n \rightarrow \infty} y_n$.

③ Prove the binomial theorem: If $a, b \in \mathbb{R}$, $m \in \mathbb{N} \Rightarrow$

$$(a+b)^m = a^m + ma^{m-1}b + \frac{m(m-1)}{1 \cdot 2} a^{m-2}b^2 + \dots + \frac{m(m-1) \dots \cdot 2 \cdot 1}{1 \cdot 2 \dots \cdot (m-1)m} b^m.$$

$$\text{E.g., } (a+b)^3 = a^3 + 3a^2b + \frac{3 \cdot 2}{1 \cdot 2} ab^2 + \frac{3 \cdot 2 \cdot 1}{1 \cdot 2 \cdot 3} b^3 \Rightarrow a^3 + 3a^2b + 3ab^2 + b^3.$$