HW 2-17 continued

③ Prove that $\lim_{n\to\infty} a^n = 0$ if $|a|<1$, while $\lim_{n\to\infty} a^n = \infty$ if $a>1$.

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① Is the sequence $b_k = \sqrt{k^2+2}$ convergent? Does it have a limit?

② Suppose $a<b<c$, $f\in C[a,b]$, $g\in C[b,c]$, and $f(b)=g(b)$. Define a function $h:[a,c]\to\mathbb{R}$ by

$$h(x)=\begin{cases} f(x) & \text{if } x\in[a,b] \\ g(x) & \text{if } x\in[b,c]. \end{cases}$$

Prove that $h\in C[a,c]$.