

HW 2-22

① Suppose $A, B \subset \mathbb{R}$, $f: B \rightarrow \mathbb{R}$, $g: A \rightarrow B$. We can form the composition $h = f \circ g$ defined by

$$h(x) = f(g(x)), \quad x \in A.$$

If g is continuous at $a \in A$ and f is continuous at $b = g(a)$, show that h is continuous at a .

② Consider a function $F: \mathbb{R} \rightarrow \mathbb{R}$, continuous at some $c \in \mathbb{R}$. If $F(x_n) = 0 \quad \forall n \in \mathbb{N}$, where $x_n \rightarrow c$, prove that $F(c) = 0$.

③ Prove that given any $z \in \mathbb{R}$, there is a sequence (q_j) of rational numbers such that $\lim_{j \rightarrow \infty} q_j = z$.