① Suppose $A, B \subset \mathbb{R}$, $f: B \to \mathbb{R}$, $g: A \to B$. We can form the composition $h = f \circ g$ defined by
$$h(x) = f(g(x)), \quad x \in A.$$

If g is continuous at $a \in A$ and f is continuous at $b = g(a)$, show that h is continuous at $\underline{a}$.

② Consider a function $F: \mathbb{R} \to \mathbb{R}$, continuous at some $c \in \mathbb{R}$. If $F(x_n) = 0$ $\forall n \in \mathbb{N}$, where $x_n \to c$, prove that $F(c) = 0$

③ Prove that given any $z \in \mathbb{R}$, there is a sequence (q_j) of rational numbers such that $\lim\limits_{j \to \infty} q_j = z$.

① Let $\varphi(x) = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \in \mathbb{R} \backslash \mathbb{Q} \end{cases}$. Find all points $c \in \mathbb{R}$ where φ is continuous.

② Let $\alpha, \beta \in \mathbb{R}$, $\alpha < \beta$, and $f: (\alpha, \beta) \to \mathbb{R}$. Suppose that f is continuous at some $c \in (\alpha, \beta)$. If $f(c) \neq 0$, prove that there is a $\eta > 0$ such that $f(x) \neq 0$ when $x \in (c - \eta, c + \eta)$. ($\eta = $ eta).

③ Give example of an unbounded function $g: [0,1] \to \mathbb{R}$ that is discontinuous only at one point.