HW 3-1

① Given a set $A \subset \mathbb{R}$ and a function $g: A \to \mathbb{R}$, construct a new function $h: A \to \mathbb{R}$,

$$h(t) = \begin{cases} g(t) & \text{if } g(t) \geq 0 \\ 0 & \text{if } g(t) < 0 \end{cases}, \quad (\text{the "positive part of } g\text{"}).$$

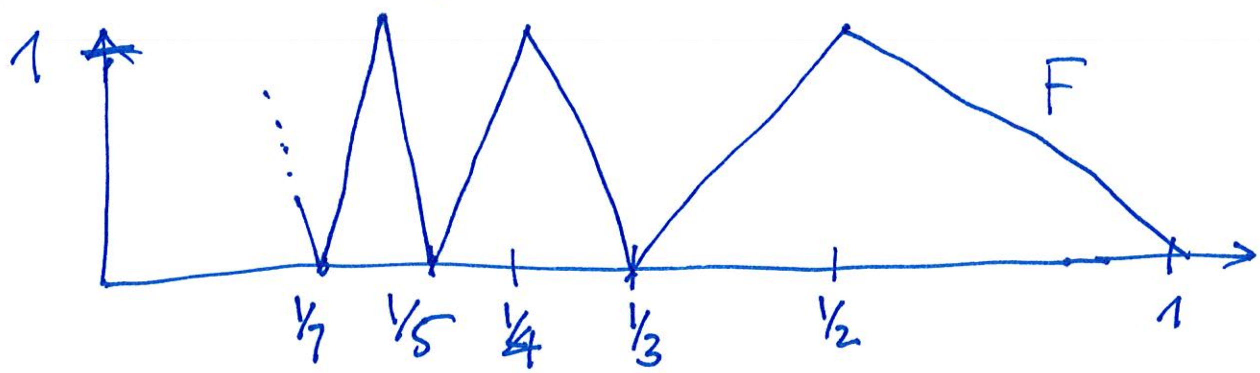
If g is continuous at $c \in A$, prove that h is also.

② Consider $\varphi: (0,\infty) \to \mathbb{R}$, $\psi: (1,\infty) \to \mathbb{R}$ given by

$\varphi(x) = 1/x \quad (x \in (0,\infty))$, $\psi(x) = 1/x \quad (x \in (1,\infty))$.

Is φ uniformly continuous? Is ψ uniformly continuous?

③ Consider the function $F: (0,1] \to \mathbb{R}$ whose graph is:



That is: $F\left(\frac{1}{n}\right) = \begin{cases} 1 & \text{if } n \in \mathbb{N} \text{ is even} \\ 0 & \text{if } n \in \mathbb{N} \text{ is odd} \end{cases}$,

and F is linear on all intervals $\left[\frac{1}{n+1}, \frac{1}{n}\right]$, $n \in \mathbb{N}$.

Is F uniformly continuous?