

HW 3-8

① Let $a < c < b$ be real numbers, $f: (a, b) \rightarrow \mathbb{R}$ an increasing function and

$$l = \sup_{(a, c)} f, \quad r = \inf_{(c, b)} f.$$

Prove that f is continuous at c if and only if $l = r$.

② If $\varphi, \psi: A \rightarrow \mathbb{R}$ are increasing functions, does it follow that $\varphi + \psi$ is increasing? That $\varphi\psi$ is increasing?

③ Should some of the answers in ② turn out to be "no", can you salvage the statement by adding extra assumptions?

HW 3-10

① If $f, g: A \rightarrow \mathbb{R}$ are discontinuous at some $c \in A$, does it follow that $f + g$ is also discontinuous at c ?

② Suppose $a, b \in \mathbb{R}$, $a < b$, and $F: [a, b] \rightarrow \mathbb{R}$ is continuous. Prove that $F([a, b]) \subset \mathbb{R}$ is also a closed interval.

③ Suppose $a, b \in \mathbb{R}$, $a < b$, and $\varphi: [a, b] \rightarrow \mathbb{R}$ is continuous. If there is a $c \in (a, b)$ such that $\varphi(c) = \max \varphi$, show that φ is not injective.