

HW 3-20

① Prove the converse direction of the Lemma in class:

If $\varphi: I \rightarrow \mathbb{R}$ is continuous at $c \in I$ and $f: I \rightarrow \mathbb{R}$ satisfies

$$f(x) = (x-c)\varphi(x) + f(c),$$

then f is differentiable at c .

② If $E: \mathbb{R} \rightarrow \mathbb{R}$ is an even function (i.e., $E(-x) = E(x) \quad \forall x \in \mathbb{R}$), and differentiable, prove that E' is an odd function (i.e., $E'(-x) = -E'(x)$).