HW 3-22

(1) At what $c \in \mathbb{R}$ is the function $g(x) = 2x + |x|$ differentiable?

(2) (a) If $p \in \mathbb{N}$, $\psi(x) = \sqrt{x^p}$ $(x > 0)$, show that ψ is differentiable, and compute its derivative.

(b) What is wrong with this "solution":

" $\psi(x) = x^{p/2} \Rightarrow \psi'(x) = \frac{p}{2} x^{\frac{p}{2}-1}$, since in class we proved $(x^n)' = n x^{n-1}$. " 2 ●

(3) Suppose $f: \mathbb{R} \to \mathbb{R}$ is differentiable at $c \in \mathbb{R}$, and $f'(c) = 0$. Prove that f cannot have an inverse function g that is differentiable at $d = f(c)$.

HW 3-24

(1) If $\varphi, \psi: (a,b) \to \mathbb{R}$ are differentiable at $c \in (a,b)$, and $\forall x \in (a,b)$ $\varphi(x) \leq \psi(x)$, while $\varphi(c) = \psi(c)$, prove $\varphi'(c) = \psi'(c)$.

(2) On what intervals is $G(x) = x^3 - 3x + 4$ increasing? decreasing? (Find maximal intervals.)

(3) Suppose $u: (a,b) \to \mathbb{R}$ is differentiable and $\forall x \in (a,b)$ $|u'(x)| \leq 1$. Prove that u is uniformly continuous.