

HW 3-3

- ① Suppose $A \subset \mathbb{R}$, $c \in \mathbb{R}$, $a_n \in A \setminus \{c\}$, and $\lim_{n \rightarrow \infty} a_n = c$.
Prove that c is a cluster point of A .
- ② Give an example of $A \subset \mathbb{R}$ whose cluster points constitute the set \mathbb{Z} .
- ③ Suppose $B \subset \mathbb{R}$ is such that every rational $c \in \mathbb{R}$ is its cluster point. Prove that then every real c is a cluster point of B .

HW 3-6

- ① If $D(x) = \begin{cases} 1 & \text{when } x \in \mathbb{Q} \\ 0 & \text{when } x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$, find $\lim_{x \rightarrow 1} D(x)$.
- ② Prove that if $A \subset \mathbb{R}$ is a finite set, any $f: A \rightarrow \mathbb{R}$ is continuous. Must f be uniformly continuous?
- ③ If $F(x) = \frac{2x^3 - 3x^2 + 1}{4x^3 + x - 6}$, $x \geq 2$, find $\lim_{x \rightarrow \infty} F(x)$.