HW 4-12

① We proved in class that if $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, then any rearrangement $\sum_{k=1}^{\infty} a_{n_k}$ is convergent (with same sum). Show that $\sum_{k=1}^{\infty} a_{n_k}$ is even absolutely convergent.

② Does the series $\sum_{j=1}^{\infty} j/2^j$ converge?

③ Does the series $\sum_{i=1}^{\infty} \frac{2^i}{2^i - 3^i}$ converge? Does it converge absolutely?

4-14

① If $g_k : A \to \mathbb{R}$ is a uniformly convergent sequence of functions, and each g_k is a bounded function, show that $\lim_{k\to\infty} g_k$ is also bounded.

② For what $t \in \mathbb{R}$ is the series $\sum_{n=1}^{\infty} \frac{1}{1+t^{2n}}$ convergent?

③ Suppose ~~$k:\mathbb{R}$~~ $\ell : (0,\infty) \to \mathbb{R}$ is a differentiable function, and $\ell'(x) = 1/x$, $\ell(1) = 0$. (We have not shown yet that there is indeed such a function, so just assume it exists.) Prove that for $x, y > 0$ then $\ell(xy) = \ell(x) + \ell(y)$.