HW 4-19

(1) Find the radius of convergence of the series

$$\sum_{k=1}^{\infty} \frac{x^k}{k^k} \quad \text{and} \quad \sum_{n=0}^{\infty} \frac{x^n}{2^n+3^n}.$$

(2) With $E(x)=\sum_{n=0}^{\infty} \frac{x^n}{n!}$, show $E(\frac{x}{2})=\sqrt{E(x)}$.

(3) Is there a power series $\sum_{n=0}^{\infty} a_n x^n$ whose sum is $|x|$ on $(-1,1)$?

HW 4-21

(1) If $a>0$, $x,y\in\mathbb{R}$, prove $a^{x+y}=a^x a^y$.

(2) For an arbitrary $k\in\mathbb{N}$, prove $\lim_{x\to\infty} \frac{E(x)}{x^k}=\infty$.

(3) Consider the function $\phi(x)=\begin{cases} e^{-1/x} & \text{if } x>0 \\ 0 & \text{if } x\le 0 \end{cases}$.

Show that ϕ is differentiable everywhere, and find $\phi'(0)$.