

HW 4-5

① If sequences of functions $f_n: S \rightarrow \mathbb{R}$ and $g_n: S \rightarrow \mathbb{R}$ converge uniformly, prove that the sequence $f_n + g_n$ also converges uniformly.

② Let $I \subset \mathbb{R}$ be an interval, $\Phi: I \rightarrow \mathbb{R}$ differentiable, $\sup_I |\Phi'| < \infty$. If A is any set, and a sequence $h_n: A \rightarrow I$ converges uniformly to a function $h: A \rightarrow I$, prove that the functions $\Phi \circ h_n: A \rightarrow \mathbb{R}$ converge uniformly to $\Phi \circ h$.

③ Show that if in the previous problem we drop the assumption $\sup |\Phi'| < \infty$, it will no longer follow that $\Phi \circ h_n$ converge uniformly.