HW 4-5

① If sequences of functions $f_n : S \to \mathbb{R}$ and $g_n : S \to \mathbb{R}$ converge uniformly, prove that the sequence $f_n + g_n$ also converges uniformly.

② Let $I \subset \mathbb{R}$ be an interval, $\Phi : I \to \mathbb{R}$ differentiable, $\sup_I |\Phi'| < \infty$. If A is any set, and a sequence $h_n : A \to I$ converges uniformly to a function $h : A \to I$, prove that the functions $\Phi \circ h_n : A \to \mathbb{R}$ converge uniformly to $\Phi \circ h$.

③ Show that if in the previous problem we drop the assumption $\sup |\Phi'| < \infty$, it will no longer follow that $\Phi \circ h_n$ converge uniformly.

HW 4-7

① Find the range of the function $B(x) = 3x^4 - 4x^3 - 6x^2 + 12x$, $x \in \mathbb{R}$, (that is, the set $\{B(x) : x \in \mathbb{R}\}$).

② Suppose $c_n \geq 0$ for all $n \in \mathbb{N}$ and the sequence (λ_n) is bounded. If $\sum_1^\infty c_n$ is convergent, show that $\sum_1^\infty \lambda_n c_n$ is also convergent.

HW 4-7 cont'd

(3) Consider two sequences $\alpha_j, \beta_j \in \mathbb{R}$, $j = 1, 2, \ldots$ such that $\{j : \alpha_j \neq \beta_j\}$ is a finite set. If $\sum_1^\infty \alpha_j$ is convergent, prove that $\sum_1^\infty \beta_j$ is also convergent.