Ratio test: Suppose $a_n \neq 0$ for $n \in \mathbb{N}$, and $\exists r \in (0,1)$ s.t. $\left|\frac{a_{n+1}}{a_n}\right| \leq r$ for all n (with finitely many exceptions). Then $\sum_1^\infty a_n$ absolutely converges.

Proof: Fix $k \in \mathbb{N}$ s.t. $\left|\frac{a_{n+1}}{a_n}\right| \leq r$ when $n = k, k+1, \dots$.
For any $n > k$ we have

$$\left.\begin{array}{l} \left|\frac{a_{k+1}}{a_k}\right| \leq r \\ \left|\frac{a_{k+2}}{a_{k+1}}\right| \leq r \\ \quad \vdots \\ \left|\frac{a_n}{a_{n-1}}\right| \leq r \end{array}\right\} \quad n-k \text{ inequalities.}$$

If we multiply them together, we obtain

$$\frac{|a_n|}{|a_k|} \leq r^{n-k}, \text{ so } |a_n| \leq a_k r^{n-k} = \frac{a_k}{r^k} r^n, \quad n > k.$$

Write $B = a_k / r^k$. Then $|a_n| \leq B r^n$ holds with finitely many exceptional n's. Since $\sum_{n=1}^\infty B r^n = B \sum_{n=1}^\infty r^n$ converges (geometric series), the comparison test implies $\sum_1^\infty |a_n|$ converges, q.e.d.