HW 1/13/23 (delayed)

① Suppose (M,ρ) is complete, $x_n \in M$, $n \in \mathbb{N}$.

(a) Prove that $\rho(x_n, x_{n+1}) < \frac{1}{2^n}$ $\forall n \Longrightarrow (x_n)$ converges.

(b) Give example that $\rho(x_n, x_{n+1}) < \frac{1}{n} \not\Longrightarrow (x_n)$ converges.

② If (M,ρ) is any metric space, the diameter of a subset $E \subset M$ is

$$\operatorname{diam} E = \sup\{\rho(x,y) : x, y \in E\} \in [0, \infty].$$

(a) Prove $\operatorname{diam} B_r(a) \leq 2r$

(b) Must even $\operatorname{diam} B_r(a) = 2r$ hold?

(Either prove that it must, or give example where it doesn't hold.)